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Landweber Iterative Method for an Inverse Source Problem of Time-space Fractional Diffusion-wave Equation

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Description

The time-space fractional diffusion-wave equation represents a fascinating and complex area of mathematical physics, capturing phenomena that standard integer-order models cannot adequately describe. This article delves into the concepts, significance, and applications of this equation without relying on the explicit use of equations or pointwise representations. Fractional calculus extends the traditional definitions of integrals and derivatives to non-integer (fractional) orders. This extension allows for a more comprehensive description of various physical processes, particularly those exhibiting anomalous diffusion or non-local behavior [1]. Unlike classical calculus, which is limited to integer-order differentiation and integration, fractional calculus introduces operators that can capture memory and hereditary properties of materials and processes.

In classical models, diffusion and wave equations describe the behavior of physical systems over time. The diffusion equation models processes such as heat conduction, where particles spread out over time, while the wave equation describes oscillatory phenomena like sound or light waves. However, these classical models assume a uniform, homogenous medium and typically do not account for complex, irregular structures or media with memory effects. The fractional diffusion-wave equation generalizes these classical models by incorporating fractional derivatives in both time and space. This generalization is crucial for modeling anomalous diffusion, where the rate of diffusion is not constant over time or space, and for capturing wave-like behavior in complex, heterogeneous media.

Fractional derivatives can be interpreted in various ways, leading to different formulations of the fractional diffusion-wave equation. The most common definitions include the Riemann-Liouville, Caputo, and Grunwald-Letnikov derivatives. These definitions differ in their treatment of initial conditions and their applicability to different types of problems. This definition is often used in theoretical studies due to its straightforward mathematical properties. However, it requires fractional initial conditions, which can be challenging to interpret physically. The Caputo derivative is more commonly used in physical applications because it allows for integer-order initial conditions, making it easier to incorporate real-world data into models. This definition provides a numerical approach to fractional differentiation, making it useful for computational purposes [2].

In many physical, biological, and financial systems, the diffusion process deviates from the classical Brownian motion. Examples include the transport of pollutants in groundwater, the movement of biological cells, and the dynamics of financial markets. The fractional diffusion-wave equation can capture these anomalous diffusion behaviors, which are characterized by non-linear scaling laws. Many materials and processes exhibit memory effects, where the current state depends on the entire history of the system. This is seen in viscoelastic materials, where stress and strain are related through a history-dependent relationship. Fractional calculus provides a natural framework for incorporating these memory effects. Real-world media are often heterogeneous and exhibit fractal-like structures. Classical models assume homogeneity and fail to capture the complexity of such media. The time-space fractional diffusion-wave equation can model the transport and wave propagation in heterogeneous and fractal media.

In physics, it is used to model anomalous transport phenomena in disordered systems, such as the movement of particles in porous media or turbulent flows. In engineering, fractional models are applied to describe the behavior of viscoelastic materials and to design systems with memory, such as control systems in robotics and aerospace engineering. Biological systems often exhibit non-standard diffusion behaviors. For example, the movement of proteins within cells or the spread of diseases can be better understood using fractional models. Finance, fractional calculus helps model the dynamics of financial markets, capturing the heavy tails and long-range dependencies observed in asset returns. In geophysics, the fractional diffusion-wave equation models the propagation of seismic waves through the Earth's heterogeneous crust, aiding in the understanding of earthquake dynamics and resource exploration [3].

These methods provide a more flexible framework for handling complex geometries and boundary conditions. They are widely used in engineering applications but require sophisticated meshing techniques. Spectral methods leverage the properties of orthogonal functions to approximate the solution. They are highly accurate for smooth problems but can be computationally expensive. These methods use probabilistic approaches to approximate the solution of fractional differential equations. They are particularly useful for high-dimensional problems and systems with random inputs. These advancements, numerical methods for fractional differential equations remain an active area of research, with ongoing efforts to improve their accuracy, efficiency, and applicability to real-world problems.

The time-space fractional diffusion-wave equation represents a powerful and versatile tool for modeling complex physical phenomena. By incorporating fractional derivatives, it captures the intricacies of anomalous diffusion, memory effects, and heterogeneous media, which classical models cannot adequately describe. The broad range of applications in physics, engineering, biology, finance, and geophysics highlights the equation's significance and utility [4,5]. As numerical methods continue to evolve, the ability to solve the time-space fractional diffusion-wave equation will improve, enabling more accurate and efficient simulations of complex systems. This progress will undoubtedly lead to new insights and advancements across various scientific and engineering disciplines, solidifying the importance of fractional calculus in modern mathematical modeling.

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Conflict of Interest

None.

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