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# Lie Algebra Techniques Using both Variational and Non-Variational Methods

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## Abstract

Numerous variations of physical issues, such as those in fluid dynamics, solid mechanics, plasma physics, quantum field theory, as well as in mathematics and engineering, include nonlinear partial differential equations. Systems of nonlinear partial differential equations have also been shown to appear in chemical and biological applications. The analytical analysis of a fully generalised (3+1)-dimensional nonlinear potential Yu-Toda-Sasa-Fukuyama equation, with applications in physics and engineering, is presented in this article. In contrast to earlier study on the problem that has already been done, the extended form of the potential Yu-Toda-Sasa-Fukuyama equation is investigated in greater detail in this paper, leading to the achievement of many novel solutions that are of interest. The nonlinear partial differential equation is fundamentally reduced to an integrable form by the use of the Lie group theory, allowing for direct integration of the outcome.

**Keywords:** Exact analytic solutions • Integrability • Variational and non-variational principles • Conserved quantities

## Introduction

Studies on nonlinear partial differential equations (NLPDEs) are crucial and important because these equations describe multiple behaviours in a variety of scientific fields, including biophysics, chemistry, fluid dynamics, plasma physics, condensed matter, biogenetics, biology, optical fibres and other engineering fields. The generalised advection-diffusion equation, a nonlinear partial differential equation in fluid mechanics that describes the motion of a buoyancy-propelled plume in a bent-on absorptive medium, is an example of this. A generalised Korteweg-de Vries-Zakharov-Kuznetsov equation was also researched in that year [1].

## Description

This equation distinguishes hot isothermal, hot adiabatic and cold immobile background species that are relevant to fluid dynamics. The crucial inclined magneto-hydrodynamic flow of an upper-convected Maxwell liquid via a leaky stretched plate was also investigated by the authors in an NLPDE that they took into account. The effects of heat production and absorption were also explored in relation to the heat transfer phenomena. It has been discovered that plasmas, which are regarded as "the most prevalent type of ordinary stuff in the universe," are connected with stars that stretch to the rarefied intracluster medium and perhaps the intergalactic regions. For instance, the authors studied a Kadomtsev-Petviashvili (KP)-Burgers-type equation with a 3 + 1 dimensional generalised variable-coefficient for a variety of cosmic dusty plasma types. In one of the cosmic or experimental dusty plasmas, this equation may represent the dust-magneto-acoustic, dust-acoustic, magneto-acoustic, positron-acoustic, ion-acoustic, ion, electron-acoustic, quantum-dust-ion-acoustic, or dust-ion-acoustic waves. In actuality, partial differential equations may be used to depict the majority of the complex processes that are regularly encountered, including diffusion, reaction, conservation, convection and many more (PDEs). Due to their fundamental nature, PDEs are extensively studied in nearly every field of study, as well as in science and engineering in particular. They are therefore the most basic models required for researching nonlinear processes [2].

It is essential to first acquire the outcomes of the regulating NLPDEs in order to comprehend these physical occurrences. The precise and approximate solutions to these problems have been secured using a variety of strange methodologies, which allows us to analyse these NLPDEs both qualitatively and quantitatively.

In addition to the aforementioned, there has been a significant increase in mathematicians and physicists discovering workable methods to find analytical solutions to NLPDEs in recent years. Some of these methods include the theory of the Lie group, the generalised Riccati equation, the complex hyperbolic function method, the sub-equation approach, the sine-cosine method, the non-classical approach, the inverse scattering transform, transformation, the F-expansion method, the extended simplest equation method, the Hirota bilinear method, the bifurcation method, the (G′/G)-expansion method, the Darboux transformation, the sine. In the late nineteenth century, Marius Sophus Lie (1842–1899) promoted the classical Lie group theory, which offers a universal tool that is highly helpful in finding symmetry groups for differential equation systems. One of the most effective, widespread and significant methods for mathematically studying PDEs is Lie symmetry analysis. It may be used to convert partial differential equations (PDEs) into ordinary differential equations (ODEs), which can then be solved in closed form. As a result, using differential equation-based mathematical models to analyse a wide range of phenomena, the group theoretical method appears to be quite effective [3].

We have the potential to address issues where it is discovered that specific physical qualities remain intact (static/constant) across time inside a single physical system thanks to the emergence of conservation laws in diverse chemical, physical and mechanical processes. Additionally, there are many applications for conservation rules, particularly in the study of PDEs. In the description of physically conserved quantities, such as momentum, mass and energy, as well as the constants of motion, for instance, they play a key role. Additionally, they may be used to determine integrability, linearization and to confirm the accuracy of numerical techniques. The conserved values for variational issues may be derived using Noether's well-known and renowned theorem. Additionally, conserved vectors may be determined using a straightforward technique for any PDE, regardless of whether it stems from a variational issue or a non-variational problem [4,5].

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It is found that the YTSF equation is a commonly used model for analysing the dynamics of solitons together with nonlinear waves that occur in fluid dynamics, plasma physics and weakly dispersive media. Think about the three-plus-one-dimensional Yu-Toda-Sasa-Fukuyama equation.

In this part, we'll first calculate the 4D-cgpYTSFe generators using the Lie group theory methodology in a methodical manner and then we'll use those results to build exact solutions to the problem we're now studying.

## Conclusion

This study effectively secures and presents a variety of classical as well as precise analytic solutions of a totally generalised (3+1)-dimensional nonlinear potential Yu-Toda-Sasa-Fukuyama Eq. In contrast to other research on the equation that has already been done, such as in, the generalised version of the potential Yu-Toda-Sasa-Fukuyama equation is studied in more detail in this paper. During the research, numerous important new solutions that have a variety of useful applications in the sciences and engineering were also discovered. The related Lie algebra was obtained using the theory of the Lie group and it was then used to the detailed reduction of the underlying equation to provide a variety of integrable and non-integrable ODEs. The Weierstrass elliptic function solution, topological kink, exponential, logarithmic and different algebraic functions that are relevant in nonlinear sciences and engineering are only a few of the intriguing solutions that were produced by integrating the ODE.

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## Conflict of Interest

No conflict of interest.

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