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On the Addition Modulus of the Aunu Pattern $\omega_i \in G_p$: An Investigation of Some Topological Properties

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Abstract

This paper investigates the topological properties on the structure generated by the addition modulus of the Aunu permutation pattern wi. The pattern wi has the generating function defined as:

$\omega i=(1\ (1+i)modp\ (1+2i)modp\ (1+3i)modp\ldots\ldots(1+((p-1)i)modp)$

For i=1,2,..$p-1$.

The numbers are arrangement on the structure $X=\{1,\ldots.,P\}$ for primes p ≥ 5.

It has been established in this paper that the generated set is a topology on X, it is not a convex set and finally, not a σ-algebra.

Keywords: Permutation pattern; Aunu numbers; Topology; Connectedness; Convexity; Sigma (σ) algebra

Introduction

Let n be a positive integer. A permutation is a bijection from the set $\{1,2,\ldots.n\}$ to itself. It maps i to $(i)\in\{1,2,\ldots.n\}$.

A pattern (classical permutation) on the other hand is a permutation ω ϵ sk [1]. Pattern is governed by specific rules. It exists in different combinatorial objects.

The inception of permutation pattern have pave way for the discovery of several mathematical structures such as symmetric group, topological group, pattern avoidance, permutation polynomials and permutation statistics. Aunu pattern a permutation generated by a certain scheme.

Aunu pattern/permutation is a partial permutation in which the first entry of every permutation is a unity (one) and its length is prime [2]. Magami, Usman and Ibrahim represented the group theoretical approach for the Aunu numbers. Catalan numbers was used to scheme for prime numbers $p \geq 5$ and $X \subseteq \mathbb{N}$ which generate the cycles of the Aunu group G_p [3,4];

$G_p=\{\omega_1, \omega_2, \ldots.\ \omega_p\}$

The group theoretical and topological properties of the Aunu numbers have also been studied, and in the Aunu pattern was represented as Γ1 non-deranged permutations. The work allows for further investigation into the behavior of the algebraic structure G_p [5-7].

Notations

Let $X=\{1,2,\ldots.p\}$ be a non-empty set of prime p ≥ 5, and $G_p=\{\omega_1,\omega_2,\ldots.\omega_{p-1}\}$ a finite group formed using ωi on an arbitrary set $X=\{1,2,\ldots.p\}$ using a permutation pattern $\omega(i)$. Then, each ωi is called a cycle [5].

Definition (power set): The power set of a set is the collection of all subsets of the set.

Definition (topology): A topology τ on a set X is the collection of subsets of X such that:

$\emptyset$ *and* X are open in τ.

1. Finite union of open members of τ is open in τ
2. Intersection of open members of τ is open in τ

The space X together with τ, (X, τ) is called a topological space.

Definition (open sets): let (X, τ) be a topological space. Then the members of τ are said to be open set.

Definition (connectedness): A topological space X is said to be disconnected if and only if there exist open subsets of X say U and V such that:

1. $U\neq\emptyset$ and $V\neq\emptyset$
2. $U \cap V = 0$
3. $U \cup V = X$

Otherwise X is said to be connected.

Definition (Cartesian product): If A and B are nonempty sets, then the Cartesian $A \times B$ is the set of all ordered pairs $(a,)$ with $a\in A$ and $b\in B$.

Definition (convex set): A set $C \subset R^n$ is said to be convex if and only if $x_1, x_2 \in \epsilon$ C the line segment defined as:

$\lambda x_1+(1-\lambda)x_2 \in C$ *where* $0 \leq \lambda \leq 1$ *and* $x_1\neq x_2$

Definition (σ- algebra): Let X be a set. A collection μ of subsets of X is called a σ-algebra if satisfies the following:

1. X ϵ μ
2. $A \in \mu \Rightarrow X \setminus A \in \mu$

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3. $A_1, A_2, \ldots, \in \mu \Rightarrow \cup^{\infty}_{i=1} A_i \in \mu$ ($A_1, A_2, \ldots$, are pairwise disjoint).

Method of Construction

This research adopts the method reported in Garba and Ibrahim (2009), in construction of the group structure G_p using Aunu pattern of permutation. The research further provides an investigation of the topological properties of the derived set. Now let

$\omega_i = (1+i)^{modp}(1+2i)^{modp}(1+3i)^{modp}\ldots\ldots(1+((p-1)i)^{modp})$

For all $p \geq 5$ and p a prime.

In constructing the subsets to be used for the study, the following definition is adopted.

$|\omega_i+\omega_j|modp$ for $i,=1,\ldots.(p-1)$ and $i \neq j$

This shall be done for all prime greater than or equal to five (5). The resulting sets from the above definition shall be considered.

Results

Theorem

Let $p \geq 5$, $X=\{1,2,\ldots.p\}$, $\omega_i,\omega_j \in Gp$ such that $i,j=\{1,\ldots.p\}$ and $i \neq j$. Then $|\omega_i+\omega_j|modp=\{X\}$ if and only if $\omega_j \neq \omega_{p-i}$ else$|\omega i+\omega j|modp=\{2\}$.

Proof: Let $p \geq 5$, $\omega_i,\omega_j \in G_p$. Then by definition

$\omega_i=(1(1+i)^{modp}(1+2i)^{modp}(1+4i)^{modp}\ldots\ldots(1+((p-1)i)^{modp})$

Clearly,

$\omega_i=\{a_1,\ldots\ldots a_p\}$ and each $a_i \in \omega_i$ is unique.

Also, let $\omega_j=\{b_1,\ldots.,b_p\}$ with each $b_i \in w_j$,

$|\omega_i+ \omega_j|modp=\{ci,\ldots cp\}=w_k \in \{1,2,\ldots p\}$

Is unique for each $c_i$and is the set is X itself.

Now, suppose $\omega_j= \omega_{p-i}$. By definition

$\omega_{p-i}=(1(1+(p-i))^{modp}(1+2(p-i)^{modp}\ldots\ldots\ldots(1+((p-1)(p-i))^{modp})$ Then,

$|\omega_i+ \omega_{p-i}|modp=\{2\}$

Therefore, for any ω_i, $\omega_j \in G_p$,

$| \omega_i+ \omega_j|modp=\{X,\{2\}$

Proposition

For any prime $p \geq 5$, and ω_i, $\omega_j \in G_p$ the set $\{X,\{2\}\}$ is the power set of X defined by the restriction$| \omega_i+ \omega j|^{modp}$.

Proof: The result is trivial by definition 1.2.1 and proves of theorem 2.1 above.

Remark: The numbers of subsets of X generated by |Proof: The result is trivial by definition 1.2.1 and prove of theorem 2.1 above.

Remark: The numbers of subsets of X generated by$|\omega_i+\omega_j|^{mopd}$ is even.

Proposition 5.3

Let $p \geq 5$, and $\omega i, \in Gp$ and $\rho_{\cup\varnothing} = \{\varnothing, X, \{2\}\}$. Then $\rho_{\cup\varnothing}$ is a toplogy on X.

Proof: let $\tau = \rho_{\cup\varnothing} = \{\varnothing, X, \{2\}\}$. From definition 1.2.2 we see that

1. $\emptyset \in \tau$ and $X \in \tau$.

$X \cup \varnothing = X \in \tau, X \cup \{2\} = X \in \tau$, and $\varnothing \cup \{2\} = \{2\} \in \tau$.

$X \cap \{2\} = \varnothing \in \tau, X \cap \{2\} = \{2\} \in \tau$, and $\varnothing \cap \{2\} = \varnothing \in \tau$.

All conditions are satisfied.

Therefore, τ is a topology on X and the collection (ω_i, τ) is a topological space.

Proposition

The space X is a connected space.

Proof: let $\tau=\{X,\{2\}\}$, by definition 1.2.4 we note that,

1. $X=\emptyset$ and $\{2\} \neq \emptyset$
2. $X \cap \{2\} = \{2\} \neq \varnothing$
3. $X \cup \{2\} = X$

Thus, no such open subsets in τ such that all three conditions are satisfied.

Proposition

Let $\tau=\{X,\{2\}\}$, and the Cartesian $\Omega=X\times\Phi$, for $\Phi =\{2\}$. Then τ is not convex.

Proof: let $\Omega=X\times\Phi$ then,

$\Omega=(x_i \times y_i)|x_i \in X$ and $yi \in \Phi.i=1,2,\ldots.$

So that,

$\Omega=\lambda x_i+(1-\lambda)x_j \times \lambda y_i+(1-\lambda)y_j$ for $0<\lambda<1$, and $I,j=1,2,\ldots.$

Clearly, by definition 3.1.6

$\lambda x_i+(1-\lambda)x_j$ for $x_i \neq x_j \in X$ is satisfied.

On the other hand,

$\lambda y_i+(1-\lambda)y_j$ is not, since $y_i=y_j=\Phi$ the singleton.

Therefore, Ω is not convex which implies τ is also not convex.

Remark: A set having a convex and a non-convex subsets is not convex.

Proposition

Let $X=\{1.,\ldots p\}$ for prime $p \geq 5$ and $\tau=\{X,\{2\}\}$ be the collection of subsets of X defined by $|\omega_i+\omega_j|modp$ where ω_i, $\omega_j \in G_p$. Then, τ is not a - algebra.

Proof: let $\tau=\{X,\{2\}\}$, by definition 1.2.7 we have;

1. $X \in \tau$
2. $X\backslash\{2\} \notin \tau$
3. $X \cup \{2\} = X \neq \varnothing$

Condition (2) and (3) are not satisfied.

Conclusion

The permutation ωi generate the set $\tau=\{X,\{2\}\}$ under the definition $|\omega_i+\omega_j|$modp where $\omega_i \neq \omega_j$ and a prime $p \geq 5$. The set τ define a topology on ωi just as the convexity of τ was shown to be void. The properties of τ shall be true for all τ={X and a singleton} subset of X.

We therefore, recommend for further restrictions on the Aunu patterns to see if more subsets of $X=\{1,\ldots.p\}$for $p \geq 5$, can be generated.

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