

Prediction of the Design Load for High Strength Concrete Columns

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Abstract

This paper presents a method to predict the strength of high strength concrete columns subjected to axial compressive load and bending moment in one direction (i.e. Uni-axial bending condition). General non-dimensional equations proposed by applying the least squares approximation and using the experimental data found in previous studies.

The results obtained from the proposed equations; showed good correlation with the tested result of square and rectangular high strength concrete columns. The proposed model is tested and applied on some columns found in previous studies and showed excellent agreement with the experimental data for high strength concrete columns.

Keywords: Column; High strength concrete; Uniaxial bending

Introduction

High strength concrete can be used to advantage in the design building since it leads to columns which can carry higher loads for the same size column cross section, or to smaller column cross section for the same size loads. Other reasons for the use of high strength concrete include its high elastic modulus, low creep deformations and improved ductility. The use of high strength concrete together with high yield strength steel reinforcement appears to be an attractive proposition for heavily loaded columns of building structures [1]. The principle reason for using high strength concrete is that it may offer that most cost-efficient solution for many structural advantageous in compression members. For this reason, the use of high-strength concrete in columns and core walls of buildings, among other applications, is increased [2].

Many studies [3-9] have demonstrated the economy of using high strength concrete in columns of high-rise buildings and low to mid rise buildings. In addition to reducing the column size, and producing a more durable material, the use of high strength concrete has been show to be advantageous with regard to lateral stiffness and axial shortening and reduction in cost of forms. There is no unique definition of high strength concrete. The Australian standard for concrete structures AS 3600-1994 [10] is limited to concrete compressive strength up to 50 Mpa, while Razvi and Soatcioglu [11], considered the strength of (41 Mpa) for normal weight concrete and (27 Mpa) for light weight concrete to be high strength concrete. This is found to be justifiable and since most of the ready-mix concrete supplied. There is no universal agreement on the applicability of ACI code requirement for calculating flexural strength of high strength concrete columns subjected to combined axial load and bending moment.

Columns are usually designed for combined for combined axial load and bending moment using the rectangular stress block shown Figure 1. This stress block was originally derived by Mattock et al. [12]. Based on the tests of un-reinforced concrete columns loaded with axial load and moments [13]. The concrete strength ranged up to 52.5 MPa parameters defined the stress block, the intensity of stress (α_1) and stress block depth ratio of the neutral axis (β_1), Mattock et al. [12] proposed:

$$\alpha_1 = 0.85$$

$$\beta_1 = 1.05 - 0.05 \left(\frac{f'_c}{6.9} \right) \leq 0.85$$

Nedderman [14], proposed a lower limit on (β_1) of 0.65 for concrete strength is excess of 55 MPa. New Zealand standard and ACI-Code recommended that the currently used parameters for the equivalent

rectangular concrete compressive stress block shown in Figure 1 are applicable up to $f'_c = 55 \text{ MPa}$. For $f'_c > 55 \text{ MPa}$ it is recommended that $\beta_1 = 0.65$ and (α_1) reduced linearly with increase in f'_c to become a minimum of (0.75) at $f'_c = 80 \text{ MPa}$.

$$\begin{aligned} \alpha_1 &= 0.85 && \text{for } f'_c \leq 55 \text{ MPa} \\ \alpha_1 &= 0.85 - 0.004(f'_c - 55) \geq 0.75 && \text{for } f'_c \leq 55 \text{ MPa} \\ \alpha_1 &= 0.75 && \text{for } f'_c = 80 \text{ MPa} \\ \beta_1 &= 0.85 && \text{for } f'_c \leq 30 \text{ MPa} \\ \beta_1 &= 0.85 - 0.008(f'_c - 30) && \text{for } f'_c \leq 30 \text{ MPa} \\ \beta_1 &= 0.65 && \text{for } f'_c \leq 55 \text{ MPa} \end{aligned}$$

Available test data indicate that typical stress-strain curves in compression for HSC are characterized by an ascending portion that is primarily linear, with maximum strength achieved at an axial strain between (0.0024 and 0.003). Therefore it may be more appropriate to use a triangular compression stress block shown in Figure 1 for HSC columns when f'_c exceeds 70 MPa intensity of compression stress equals ($\alpha_1 = 0.63$); rather than $0.85 f'_c$ or ($\alpha_1 = 0.63$); and the depth of the rectangular compression block is equal to $\alpha_1 = 0.67c$ or $\beta_1 = 0.67$.

The Canadian Code [15]; suggested the following modified rectangular stress block:

$$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67$$

Ibrahim et al. [16], compared the concrete component of the measured load and moment strength of (94) tests of eccentrically loaded columns with concrete strengths ranging up to 130 MPa and they conclude that the max. Concrete strain before spalling were greater than (0.003), and the HSC columns can be designed based on rectangular stress block with some modification of the parameters as below:

$$\alpha_1 = 0.85 - 0.00125 f'_c \geq 0.725$$

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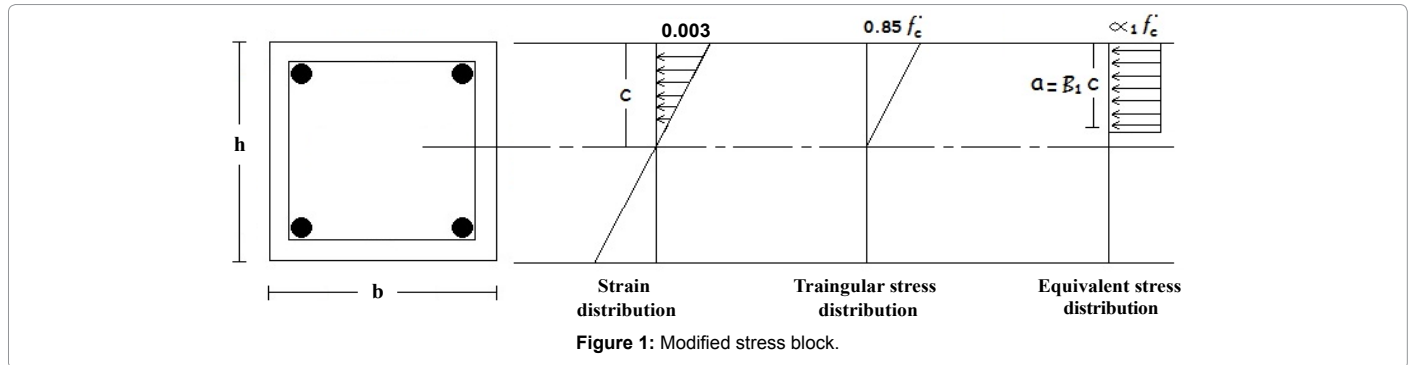


Figure 1: Modified stress block.

$$\beta_1 = 0.85 - 0.0025f'_c \geq 0.6$$

Rangan and Lioyd [2]; presented a program on the behaviour and strength of HSC columns under eccentric compression, they developed a theory to predict the load-deflection behaviour and the failure load, based on a simplified stability analysis and a stress-strain relation of strength concrete in compression the average ratio of test failure load to predicted failure load was (1.13) with coefficient of variation of (10%).

Saatcioglu and Razvi [11], investigated the strength and deformability of confined high strength concrete columns based on available experimented data up to 250 columns, tested either under monotonically increasing concentric loading or reversed lateral loading were evaluated in terms of load, ductility and draft capacities. The results indicate that the confinement requirement for HSC columns significantly more stringent than those for NSC columns and it is possible to obtain ductile behaviour in HSC columns through proper confinement. The use of high strength confinement steel reduces the need to impose unrealistically high volumetric ratios to attain deformability usually expected of NSC columns. The results also indicate that the product of the volumetric ratio and strength of confinement steel, normalized with respect to concrete strength, can be used as a design parameter.

Setty and Rangan [17], presented a study on eccentrically loaded HSC columns, they proposed a modified method to predict the failure load of eccentrically loaded reinforced concrete columns using an equivalent rectangular stress block that applies to all grades of concrete. The calculated failure load correlates well with the failure load to predict of (143) test columns, the mean value of test failure load to predict failure load was (1.08) with coefficient of variation of (12%). Fafitis and Shah [18], proposed an analytical expressions for the stress strain curves of confined and unconfined high strength concrete. Also moment-curvature relationships were predicted for columns subjected to reversed lateral loading. The analysis and design based on the assumptions of the ACI Code [19] and principles of static and graphs of interaction diagram for short columns subjected to bending moments and axial compression load. The detailing of the design and analysis equations are found in many text books of reinforced concrete design [20-26]. In this paper, general equations containing non-dimensional terms in the form of second and fourth degree polynomial are proposed. The equations have been used to analysis and design of high strength reinforced concrete short and tied columns subjected to uniaxial bending moment and axial load.

Theory and Analysis

Two general equations of 2nd and 4th degree were proposed to present the non-dimensional interaction surface in the following form:

$$Z = K_0 + K_1X + K_2Y + K_3XY + K_4X^2 + K_5Y^2 \quad (1)$$

and

$$Z = K_0 + K_1X + K_2Y + K_3XY + K_4X^2Y + K_5XY^2 + K_6X^2Y^2 \quad (2)$$

Where

$$X = M_{nx} / M_{nox} \quad \text{or} \quad X = M_{nx} / M_{nbx}$$

$$Y = M_{ny} / M_{noy} \quad \text{or} \quad Y = M_{ny} / M_{nby}$$

$$Z = (P_n - P_{nb}) / (P_{no} - P_{nb}) \quad \text{or} \quad Z = p_n / p_{n0}$$

P_n = Nominal normal axial load (kN)

P_{nb} = Nominal axial strength at balance condition (kN)

P_{no} = Nominal normal compression load capacity of the column (kN)

M_{nx} = Nominal bending moment in x-direction (kN.m)

M_{nox} = Pure bending moment in x-direction (kN.m)

M_{nbx} = Nominal bending moment in x-direction (kN.m) at balance condition.

M_{ny} = Nominal bending moment in y-direction (kN.m)

M_{noy} = Pure bending moment in y-direction (kN.m)

M_{nby} = Nominal bending moment in y-direction (kN.m) at balance condition.

and ($K_0, K_1, K_2, \dots, K_6$) are coefficients.

The coefficients of the equivalent rectangular stress block used in the analysis are shown below:

$$\alpha_1 = 0.85 \quad \text{for } f'_c < 55 \text{ MPa}$$

$$\alpha_1 = 0.85 - 0.004(f'_c - 55) \quad \text{for } 80 > f'_c > 80 \text{ MPa}$$

$$\alpha_1 = 0.85 - 0.00125f'_c \quad \text{for } 100 > f'_c > 80 \text{ MPa}$$

$$\alpha_1 = 0.75 \quad \text{for } f'_c > 100 \text{ MPa}$$

$$\beta_1 = 0.85 \quad \text{for } f'_c > 28 \text{ MPa}$$

$$\beta_1 = 0.85 - 0.05 \left(\frac{f'_c - 28}{6.9} \right) \quad \text{for } f'_c > 28 \text{ MPa}$$

$$\beta_1 = 0.65 \quad \text{for } f'_c > 55 \text{ MPa}$$

The least squares approximations [27] were applied to minimize the error function (S) with respect to the coefficients:

$$S = \sum_1^N (Z_c - Z_e)^2 \quad (3)$$

Where Z_c = Values given by the proposed equation.

Z_e = Values from the test results.

S = Sum of errors.

N = No. of data.

Considering equation (1):

$$\sum_1^N [K_0 + k_1X + K_2Y + K_3XY + K_4X^2 + K_5Y^2 - Z_e]^2 \quad (4)$$

Minimization with respect to the coefficients ($K_0, K_1, K_2 + K_3 + K_4$ & K_5) the following equation is obtained:

$$\begin{bmatrix} N & \sum X & \sum Y & \sum XY & \sum X^2 & \sum Y^2 \\ \sum X & \sum X^2 & \sum XY & \sum X^2Y & \sum X^3 & \sum XY^2 \\ \sum Y & \sum XY & \sum Y^2 & \sum XY^2 & \sum X^2Y & \sum Y^3 \\ \sum XY & \sum X^2Y & \sum XY^2 & \sum X^2Y^2 & \sum X^3Y & \sum XY^3 \\ \sum X^2 & \sum X^3 & \sum Y & \sum X^2Y & \sum X^4 & \sum X^2Y^2 \\ \sum Y^2 & \sum XY^2 & \sum Y^3 & \sum XY^3 & \sum X^2Y & \sum Y^4 \end{bmatrix} \begin{bmatrix} K_0 \\ K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \end{bmatrix} = \begin{bmatrix} \sum Ze \\ \sum XZe \\ \sum YZe \\ \sum XYZe \\ \sum X^2Ze \\ \sum Y^2Ze \end{bmatrix} \quad (5)$$

First equation (5) is solved by computer using experimental data shown in Tables 1 and 2 for normal and high strength concrete columns to show the accuracy of the proposed model.

The nominal bending moment at balance strain condition and pure bending condition are calculated for each test about X and Y axes ($M_{nbx}, M_{nox}, M_{nby}$ and M_{noy}) also the maximum axial strength (P_{no}) and nominal axial strength at balance strain condition (P_{nb}) are also determined for each test. Based on these data, the value of ($X=Mnx/Mnox$ or $X=Mnx/Mnbx$), ($Y=Mny/Mnoy$ or $Y=Mny/Mnby$) and ($Z=(Pn-Pnb)/(Pno-Pnb)$ or $Z=Pn/Pno$) are determined.

Substituting the values of X, Y and Z in equation (5) and solving this equation, the value of the coefficients ($K_0, K_1, K_2, \dots K_6$) are determined. Computers programs are prepared to perform all these calculations.

Pure bending moment condition

$$X = \frac{M_{nx}}{M_{nbx}}; Y = \frac{M_{ny}}{M_{nby}}; Z = \frac{P_n}{P_{no}}$$

$$Z = 0.959 - 0.114X - 1.242Y - 0.106XY - 0.039X^2 + 0.507Y^2 \quad (6)$$

and

$$Z = 0.986 - 0.21X - 0.652Y - 1.546XY - 0.483X^2Y + 1.149XY^2 - 0.105X^2Y^2 \quad (7)$$

Balance bending moment condition

$$X = \frac{M_{nx}}{M_{nbx}}; Y = \frac{M_{ny}}{M_{nby}}; Z = \frac{P_n}{P_{no}}$$

$$Z = 0.974 - 0.283X - 1.497Y + 0.012XY - 0.11X^2 + 0.7316Y^2 \quad (8)$$

and

$$Z = 0.99 - 0.412X - 0.868Y - 0.913XY - 0.62X^2Y + 0.84XY^2 + 1.442X^2Y^2 \quad (9)$$

Equation (1) can be re-arranged in the following form for the purpose of solution:

$$AP_n^2 + BP_n + C = 0 \quad (10)$$

Balance bending moment condition

$$A = \frac{K_3e_xe_y}{M_nbx.M_nby} + \frac{K_4e_y^2}{M_nbx^2} + \frac{K_5e_x^2}{M_nby^2}$$

$$B = \frac{K_1e_y}{M_{nox}} + \frac{K_2e_x}{M_{noy}} - \frac{1}{P_{no} - P_{nb}}$$

$$C = K_0 + \frac{P_{nb}}{P_{no} - P_{nb}}$$

Pure bending moment condition

$$A = \frac{K_3e_xe_y}{M_{nox}.M_{noy}} + \frac{K_4e_y^2}{M_{nox}^2} + \frac{K_5e_x^2}{M_{noy}^2}$$

$$B = \frac{K_1e_y}{M_{nbx}} + \frac{K_2e_x}{M_{nby}} - \frac{1}{P_{no} - P_{nb}}$$

$$C = K_0 + \frac{P_{nb}}{P_{no} - P_{nb}}$$

Finally, equation (10) can be solved easily by computer or by hand calculations:

In case

$$Z = \frac{P_n}{P_{no}}; B = \frac{K_1e_y}{M_{nbx}} + \frac{K_2e_x}{M_{nby}} - \frac{1}{P_{no}}; C = K_0$$

Also equation (2) re-arranged in the following form for the purpose of solution:

$$AP_n^4 + BP_n^3 + CP_n^2 + DP_n + E = 0 \quad (11)$$

$$A = k_6 \left(\frac{e_y}{M_{nox}} \right)^2 \left(\frac{e_x}{M_{noy}} \right)^2$$

$$B = K_4 \left(\frac{e_y}{M_{nox}} \right)^2 \left(\frac{ex}{M_{noy}} \right) + K_5 \left(\frac{e_y}{M_{nox}} \right)^2 \left(\frac{ex}{M_{noy}} \right)^2$$

$$C = K_3 \left(\frac{e_y}{M_{nox}} \right) \left(\frac{e_x}{M_{noy}} \right)$$

$$D = K_1 \left(\frac{e_y}{M_{nox}} \right) + K_2 \left(\frac{e_y}{M_{noy}} \right) - \frac{1}{P_{no} - P_{nb}}$$

$$E = K_0 + \frac{P_{nb}}{P_{no} - P_{nb}}$$

In case using balance strain condition; use the moments (M_{nbx} and M_{nby}) instead of (M_{nox} and M_{noy}) respectively.

Because the 4th degree equation is more complicated for solution and results of equations (6-9) which are represent normal and high concrete strength columns subjected to uniaxial and biaxial bending conditions give results relatively far from the experimental data, so the study concentrate on the high strength concrete columns with uniaxial bending condition. And 2nd degree equation is selected for this purpose [27-30].

Pure bending moment condition

$$X = \frac{M_n}{M_{no}}; Z = \frac{P_n - P_{nb}}{P_{no} - P_{nb}}$$

$$Z = 0.929 - 0.044X - 0.064X^2 \quad (12)$$

Balance bending moment condition

$$X = \frac{Mn}{Mnb}; Z = \frac{Pn - Pnb}{Pno - Pnb}$$

No.	Ref.	width (in)	Depth (in)	f'_c (psi)	f'_y (ksi)	Reinforcement	Eccen. e_x (in)	Eccen. e_y (in)	P_u (exp) kips	P_{test}/P_{cal}			
										EQ6	EQ7	EQ8	EQ9
1	[29,30]	4	4	5435	45.6	4 # 4	2.82	2.82	13.5	1.61	1.85	0.89	0.88
2		4	4	5435	45.6	4 # 4	2.82	2.82	14.3	2.01	2.15	0.88	0.89
3		6	8	3200	53.5	4 # 5	3	4	32	1.04	1.01	0.61	0.58
4		6	8	3700	53.5	4 # 5	6	8	17	3.43	3.06	1.61	1.53
5		6	8	3500	53.5	4 # 5	6	4	21	2.35	1.83	1.31	1.23
6		6	8	3800	53.5	4 # 5	3	8	24	1.69	2.43	1.08	1.38
7		4	4	3200	44.5	4 # 3	1	1.5	21	0.6	0.45	0.15	0.19
8		4	4	4095	44.5	4 # 3	1	1.5	24.8	0.08	0.05	0.03	0.04
9		4	4	3905	73	9 Φ 1/4 in	2.5	3.5	9.6	0.42	0.56	0.65	0.9
10		4	4	3806	73	9 Φ 1/4 in	3	3.5	8.7	0.52	0.65	0.79	0.91
11		4	4	3894	73	9 Φ 1/4 in	3.5	3.5	8				
12		4	4	3830	73	9 Φ 1/4 in	2	2	14.3				
13		4	4	3715	73	9 Φ 1/4 in	0.5	5.5	10.8				
14		4	4	3895	73	9 Φ 1/4 in	0.5	7	6.24	0.68	0.8	0.95	0.98
15		4.25	4.25	3545	44.5	8 # 3	3	2	13.9	1.02	0.95	1.05	0.94
16		4.25	4.25	3884	44.5	8 # 3	3.25	2.25	11.8	1.07	1.06	1.06	0.98
17		4.25	4.25	4227	44.5	8 # 3	2.5	3	13.6				
18		8	8	4230	46.8	8 # 5	0.8282	3.091	141.4	1.07	1.36	1.18	1.38
19		8	8	3735	46.8	8 # 5	0.764	1.848	173.5				
20		8	8	4860	46.8	8 # 5	2	3.464	120	0.68	0.76	0.98	1.21
21		8	8	4635	46.8	8 # 5	2.5	4.33	89	0.39	0.48	0.75	1.19
22		8	8	2805	46.8	8 # 5	1.414	1.414	134.5				
23		8	8	3998	46.8	8 # 5	2.546	2.546	112.5				
24		8	8	4275	47.6	8 # 5	2.828	2.828	116	0.8	0.8	1.09	1.07
25		8	8	4950	46.8	8 # 5	4	4	83.125	0.83	0.95	1.91	1.81
26		6	9	4590	46.8	8 # 5	0.9987	1.498	176.5				
27		6	9	3690	46.8	8 # 5	2.194	3.328	90	1.21	1.3	1.75	1.79
28		6	9	3548	46.8	8 # 5	2.992	4.493	70	0.29	0.33	1.2	0.64
29		6	9	3645	46.8	8 # 5	1.273	1.273	153	1.45	1.38	2.14	1.97
30		6	9	4482	46.8	8 # 5	3.182	3.182	85	0.4	0.37	0.88	0.52
31		6	9	3485	46.8	8 # 5	3.117	1.8	90				
32		6	12	3402	46.8	8 # 5	2.364	4.472	104.5	0.51	0.43	31.55	0.64
33		6	12	3105	46.8	8 # 5	3	6	70	0.56	0.65	1.36	1.21
34		6	12	4023	47.8	8 # 5	3.394	3.394	98	0.81	0.67	1.27	1.02
35		6	12	3800	71.6	8 # 5	2.598	1.5	122	0.46	0.37	0.54	0.8
36		5	5	4633	71.6	4 # 4	0.4087	0.9867	73	0.8	0.91	0.85	0.92
37		5	5	4633	71.6	4 # 4	0.4076	0.9839	77	0.9	1.06	0.97	1.08
38		5	5	4997	71.6	4 # 4	2.624	1.087	38	0.88	0.74	0.67	0.64
39		5	5	4997	71.6	4 # 4	2.624	1.087	35.8	0.62	0.53	0.48	0.44
40		5	5	4997	71.6	4 # 4	4.89	2.025	19.1	0.47	0.4	0.37	0.33
41		5	5	4997	71.6	4 # 4	5.028	2.083	17.6	0.52	0.44	0.42	0.37
42		5	5	4633	71.6	4 # 4	0.7623	0.7628	78.2	1.68	1.87	1.62	1.61
43		5	5	4633	71.6	4 # 4	0.7545	0.7545	75.5	1.46	1.59	1.41	1.38
44		5	5	5164	71.6	4 # 4	1.897	1.897	38.7	0.7	0.8	0.59	0.61
45		5	5	5164	71.6	4 # 4	1.947	1.947	37	0.57	0.66	0.48	0.5
46		5	5	5164	71.6	4 # 4	3.784	3.784	18.5	0.48	0.56	0.41	0.43
47		5	5	5164	71.6	4 # 4	3.725	3.725	18.9	0.46	0.54	0.4	0.41
48		5	5	3480	71.6	4 # 4	2.505	2.505	42.1	2.84	2.14	2.67	3.23
49		5	5	3480	71.6	4 # 4	4.889	2.025	18.5	1.59	1.73	2.11	1.97
50		5	5	3660	71.6	4 # 4	1.917	1.917	38.2	1.14	3.59	1.12	1.13
51		5	5	3660	71.6	4 # 4	3.712	3.712	18.2	0.8	0.77	0.69	0.63
52		10	10	5100	43.6	8 # 7	12.5	0	88	0.39	3.56	0.43	0.91
53		6	8	3700	53.5	4 # 5	6	0	24	0.38	0.4	0.42	0.43
54		6	8	3900	53.5	4 # 5	3	0	60	1.67	1.92	1.27	0.98
55		6	8	3700	53.5	4 # 5	0	4	70	0.91	2.27	0.82	0.72

56		6	8	4600	53.5	4 # 5	0	8	32	0.99	0.62	1.11	1.12
57		4	4	3426	44.5	4 # 3	5	0	6.445	1.09	0.71	1.18	1.43
58		4	4	3426	44.5	4 # 3	3	0	11.91	0.91	0.57	1.03	0.91
59		6	6	3000	40	8 # 4	2	0	72	1.04	0.65	1.2	1.18
60		6	6	3400	40	8 # 4	2	0	80	1.04	1.01	1.03	1.01
61		6	6	4200	40	8 # 4	2	0	100	1	1.01	0.97	0.97
62		6	6	4300	40	8 # 4	2	0	106	1.01	1.02	0.98	0.98
63		5	5	4688	71.6	4 # 4	1.031	0	78	0.99	0.99	0.97	0.97
64		5	5	4688	71.6	4 # 4	1.06	0	81.5	0.99	1.01	0.95	0.96
65		5	5	5376	71.6	4 # 4	2.8	0	42.3	0.96	0.98	0.93	0.94
66		5	5	5376	71.6	4 # 4	2.725	0	46	1.06	1.03	1.04	1.03
67		5	5	5376	71.6	4 # 4	5.25	0	23.6	1.05	1.02	1.04	1.02
68		5	5	5376	71.6	4 # 4	5.24	0	23.7	1.01	1.04	0.97	0.99
69		5	5	3666	71.6	4 # 4	2.73	0	42.6	1.56	0.97	1.37	1
70		5	5	3666	71.6	4 # 4	5.275	0	19.7	1.81	1.14	1.59	1.17
Average P_{test}/P_{cal} = Variance										1.1	1.32	1.44	1.37
										0.131	0.13	0.13	0.13

Table 1: Experimental data for Normal strength concrete columns.

No.	Ref.	width (mm)	Depth (mm)	f'_c (M Pa)	f'_y (M Pa)	Reinforcement	Eccen. (e) mm	P_u (exp) kN	P_{cal} (eq12) kN	P_{cal} (eq13) kN	P_{test}/P_{cal} =					
											EQ6	EQ7	EQ8	EQ9	EQ12	EQ13
1	[17]	100	300	53.1	454	6 Φ 12	10	1387	1506.43	1463.26	0.58	0.59	0.52	0.55	0.92	0.95
2		100	300	54.2	454	4 Φ 12	10	1200	1432.35	1365.03	0.70	0.96	0.67	0.87	0.84	0.88
3		100	300	56.6	454	4 Φ 12	10	1375	1475.09	1402.11	1.12	1.29	1.08	2.21	0.93	0.98
4		100	300	67.1	454	6 Φ 12	10	1464	1732.09	1668.28	0.88	0.89	0.88	0.89	0.85	0.88
5	[2]	175	175	58	454	6 Φ 12	15	1476	1467.59	1225.69	0.76	0.77	0.74	0.75	1.01	1.20
6		175	175	58	454	6 Φ 12	50	830	948.49	584.02	0.90	0.91	0.88	0.88	0.88	1.42
7		175	175	58	454	6 Φ 12	65	660	807.32	472.27	0.78	0.78	0.77	0.77	0.82	1.40
8		100	300	58	454	6 Φ 12	10	1192	1596.56	1545.52	1.03	1.07	1.09	1.12	0.75	0.77
9		175	175	58	454	4 Φ 12	15	1140	1336.46	1058.35	0.67	0.67	0.80	0.76	0.85	1.08
10		175	175	58	454	4 Φ 12	50	723	792.69	467.29	0.47	0.47	0.57	0.53	0.91	1.55
11		175	175	58	454	4 Φ 12	65	511			0.64	0.64	0.64	0.64		
12		100	300	58	454	4 Φ 12	10	915	1497.26	1421.50	0.78	0.81	0.77	0.79	0.61	0.64
13		175	175	92	454	4 Φ 12	15	1704	1883.83	1509.86	0.66	0.62	0.60	0.58	0.90	1.13
14		175	175	92	454	4 Φ 12	50	1018	1138.76	677.29	0.44	0.44	0.74	0.73	0.89	1.50
15		175	175	92	454	4 Φ 12	65	795	956.54	543.61	0.86	0.90	0.71	0.71	0.83	1.46
16		100	300	92	454	4 Φ 12	10	1189	2088.53	1981.78	0.63	0.61	0.39	0.40	0.57	0.60
17		175	175	92	454	4 Φ 12	15	1745	1726.97	1310.30	0.39	0.38	0.37	0.36	1.01	1.33
18		175	175	92	454	4 Φ 12	50	908	959.68	549.79	0.38	0.38	0.72	0.69	0.95	1.65
19		175	175	92	454	4 Φ 12	65	663	794.42	437.86	1.05	1.09	0.77	0.77	0.83	1.51
20		100	300	92	454	4 Φ 12	10	1043	1980.99	1830.33	0.67	0.58	0.32	0.33	0.53	0.57
21		175	175	97	454	4 Φ 12	15	1975	1791.96	1354.72	1.24	1.29	1.54	1.51	1.10	1.46
22		175	175	97	454	4 Φ 12	50	1002	990.51	566.19	0.92	0.72	0.37	0.38	1.01	1.77
23		175	175	97	454	4 Φ 12	65	746	819.24	450.72	0.40	0.33	0.27	0.26	0.91	1.66
24		100	300	97	454	4 Φ 12	10	1610	2057.47	1892.52	0.67	0.68	0.64	0.64	0.78	0.85
25		175	175	97	454	4 Φ 12	15	1932	1791.96	1354.72	1.19	1.23	1.07	1.10	1.08	1.43
26		175	175	97	454	4 Φ 12	50	970	990.51	566.19	0.78	0.64	0.50	0.49	0.98	1.71
27		175	175	97	454	4 Φ 12	65	747	819.24	450.72	0.40	0.33	0.26	0.25	0.91	1.66
28		100	300	97	454	4 Φ 12	10	1650	2057.47	1892.52	0.70	0.71	0.67	0.67	0.80	0.87
29	[28]	200	200	103	576	8 Φ 16	0	3983			1.07	1.04	1.05	1.04		
30	Series A	200	200	103	576	8 Φ 16	5	3974	3578.90	3510.31	1.09	1.09	1.10	1.11	1.11	1.13
31		200	200	103	576	8 Φ 16	10	3476	3471.93	3247.12	1.01	1.03	1.05	1.08	1.00	1.07
32		200	200	103	576	8 Φ 16	20	3073	3193.02	2637.24	0.97	1.01	1.09	1.12	0.96	1.17
33		200	200	103	576	8 Φ 16	20	3068	3193.02	2637.24	0.96	1.00	1.09	1.12	0.96	1.16
34		200	200	103	576	8 Φ 16	0	4110			1.11	1.08	1.09	1.07		
35		200	200	103	576	8 Φ 16	5	3861	3578.90	3510.31	1.08	1.08	1.10	1.11	1.08	1.10

36		200	200	103	576	8 Φ16	10	3176	3471.93	3247.12	0.90	0.91	0.93	0.95	0.91	0.98
37		200	200	103	576	8 Φ16	10	3990	3471.93	3247.12	1.20	1.23	1.27	1.30	1.15	1.23
38		200	200	103	576	8 Φ16	10	3606	3471.93	3247.12	1.05	1.08	1.11	1.13	1.04	1.11
39		200	200	103	576	8 Φ16	20	3019	3193.02	2637.24	0.94	0.98	1.06	1.09	0.95	1.14
40	[28]	200	200	101	576	8 Φ16	0	3438			0.92	0.89	0.91	0.89		
41	Series R	200	200	101	576	8 Φ16	10	2998	3423.04	3206.30	0.85	0.86	0.88	0.90	0.88	0.94
42		200	200	101	576	8 Φ16	20	2776	3152.86	2612.36	0.85	0.88	0.95	0.97	0.88	1.06
43		200	200	101	576	8 Φ16	30	2483	2859.05	2134.18	0.81	0.85	0.97	0.99	0.87	1.16
44		200	200	101	576	8 Φ16	50	1958	2339.31	1527.81	0.69	0.72	0.95	0.91	0.84	1.28
45		200	200	101	576	8 Φ16	0	3326			0.89	0.86	0.87	0.86		
46		200	200	101	576	8 Φ16	10	3002	3423.04	3206.30	0.85	0.86	0.88	0.90	0.88	0.94
47		200	200	101	576	8 Φ16	20	2643	3152.86	2612.36	0.79	0.82	0.87	0.90	0.84	1.01
48		200	200	101	576	8 Φ16	30	2456	2859.05	2134.18	0.91	0.95	1.11	1.12	0.86	1.15
49		200	200	101	576	8 Φ16	50	2111	2339.31	1527.81	0.81	0.83	1.19	1.09	0.90	1.38
50		200	200	101	576	8 Φ16	80	1434	1792.88	1059.02	0.52	0.52	0.84	0.72	0.80	1.35
51	[28]	200	200	92	560	8 Φ12	0	3383			0.84	0.82	0.83	0.82		
52	Series J	200	200	92	560	8 Φ12	10	2626	2731.06	2421.12	1.01	0.98	0.99	0.98	0.96	1.08
53		200	200	92	560	8 Φ12	20	2878	2386.26	1796.40	1.08	1.12	1.08	1.11	1.21	1.60
54		200	200	92	560	8 Φ12	0	2959			1.29	1.34	1.29	1.31		
55		200	200	92	560	8 Φ12	10	3091	2731.06	2421.12	1.30	1.30	1.28	1.27	1.13	1.28
56		200	200	92	560	8 Φ12	20	2784	2386.26	1796.40	1.19	1.20	1.18	1.18	1.17	1.55
57	[28]	200	200	87	560	8 Φ12	0	2541			1.02	0.99	1.00	0.98		
58	Series Z	200	200	87	560	8 Φ12	0	2946			1.11	1.15	1.11	1.14		
59		200	200	87	560	8 Φ12	10	2762	2635.03	2351.50	1.54	1.59	1.54	1.56	1.05	1.17
60		200	200	87	560	8 Φ12	20	2628	2316.65	1761.46	1.18	1.18	1.16	1.16	1.13	1.49
61		200	200	87	560	8 Φ12	30	2238	2012.10	1372.29	1.14	1.10	1.12	1.10	1.11	1.63
62		200	200	87	560	8 Φ12	30	2160	2012.10	1372.29	0.96	0.99	0.95	0.97	1.07	1.57
63		200	200	87	560	8 Φ12	0	2962			1.46	1.50	1.42	1.44		
64		200	200	87	560	8 Φ12	10	3000	2635.03	2351.50	0.97	0.94	0.95	0.94	1.14	1.28
65		200	200	87	560	8 Φ12	20	2886	2316.65	1761.46	1.21	1.26	1.20	1.23	1.25	1.64
66		200	200	87	560	8 Φ12	30	2145	2012.10	1372.29	1.37	1.41	1.33	1.36	1.07	1.56
67	[28]	200	200	87	560	8 Φ12	0	2586			0.86	0.84	0.85	0.83		
68	Series M	200	200	87	560	8 Φ12	0	3162			1.10	1.07	1.08	1.06		
69		200	200	87	560	8 Φ12	10	2998	2635.03	2351.50	1.22	1.26	1.22	1.25	1.14	1.27
70		200	200	87	560	8 Φ12	20	2793	2316.65	1761.46	1.45	1.49	1.44	1.47	1.21	1.59
71		200	200	87	560	8 Φ12	30	2176	2012.10	1372.29	1.22	1.22	1.20	1.19	1.08	1.59
72		200	200	87	560	8 Φ12	50	1887	1549.78	938.27	2.19	1.58	1.93	1.52	1.22	2.01
73		200	200	87	560	8 Φ12	0	3061			1.06	1.03	1.04	1.02		
74		200	200	87	560	8 Φ12	0	3758			1.34	1.31	1.32	1.30		
75		200	200	87	560	8 Φ12	5	3428	2765.45	2672.32	1.31	1.33	1.31	1.32	1.24	1.28
76		200	200	87	560	8 Φ12	10	3263	2635.03	2351.50	1.37	1.43	1.38	1.41	1.24	1.39
77		200	200	87	560	8 Φ12	20	2884	2316.65	1761.46	1.54	1.58	1.54	1.56	1.24	1.64
78		200	200	87	560	8 Φ12	20	2943	2316.65	1761.46	1.61	1.65	1.60	1.61	1.27	1.67
79		200	200	87	560	8 Φ12	0	3137			1.09	1.06	1.07	1.05		
80		200	200	87	560	8 Φ12	0	3927			1.41	1.37	1.39	1.37		
81		200	200	87	560	8 Φ12	0	3856			1.38	1.35	1.36	1.34		
82		200	200	87	560	8 Φ12	5	3484	2765.45	2672.32	1.34	1.36	1.33	1.35	1.26	1.30
83		200	200	87	560	8 Φ12	10	3243	2635.03	2351.50	1.36	1.42	1.36	1.40	1.23	1.38
84		200	200	87	560	8 Φ12	20	2890	2316.65	1761.46	1.55	1.59	1.54	1.56	1.25	1.64
Average $P_{test}/P_{cal} =$ Variance											1.1	1.315	1.44	1.374	0.985	1.282
											0.131	0.131	0.131	0.131	0.029	0.093

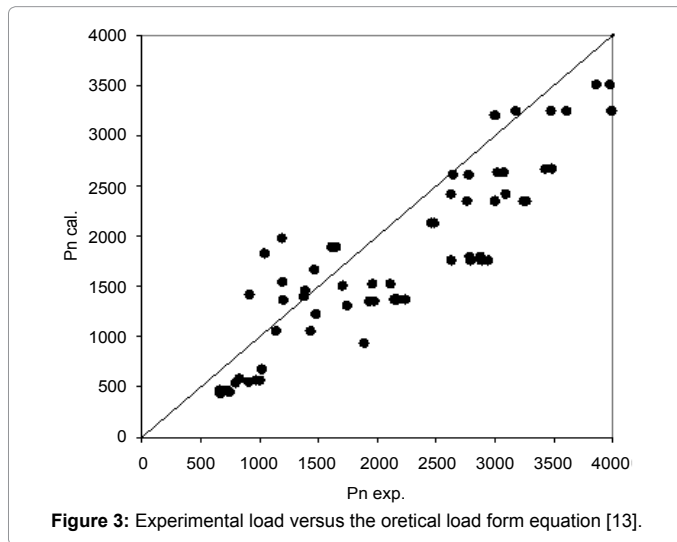
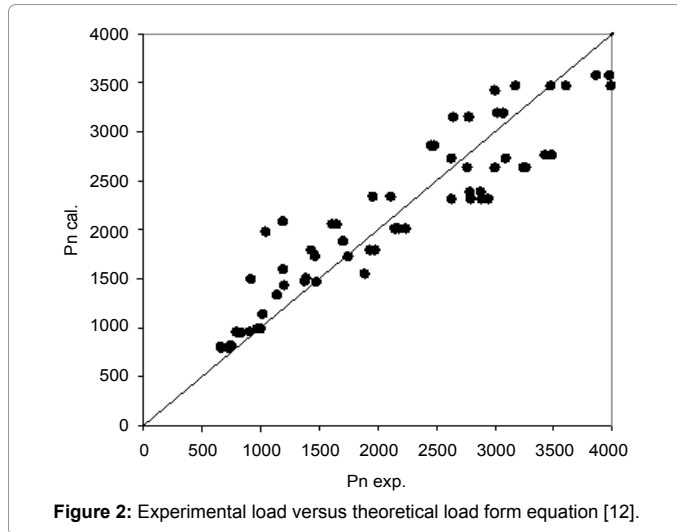
Table 2: Experimental data for High strength concrete columns.

$$Z=0.92 + 0.007X - 0.326 X^2 \quad (13)$$

In general, the results obtained from equations (12) and (13) are found in good agreement with the experimental load for high strength

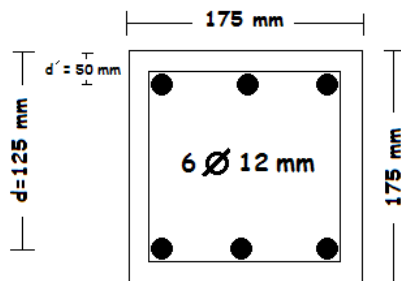
concrete columns subjected to uni-axial bending condition, as shown in Figures 2 and 3.

The following examples explain the calculations and application of the proposed method.



Example1: (specimen IA/ref. [2]).

Experimental data:



$$f'_c = 58MPa$$

$$f_y = 454MPa$$

Eccentricity (e) = 15 mm

$P_n \text{ exp.} = 1476 \text{ kN}$

Solution:

$$\alpha_1 = 0.85 - 0.004(58 - 55) = 0.838$$

$$\beta_1 = 0.65(f'_c > 55MPa)$$

Pure bending

$$\sum F_x = 0$$

$$(\alpha_1 f'_c)(\beta_1 c)b + A'_s f'_s - A_s f_y = 0$$

$$0.838(58)(0.65c)(175) + 339.3(600 \frac{c-50}{50}) - 339.3(454) = 0$$

Simplify to:

$$c^2 + 8.96c - 1841.118 = 0$$

$$C = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8.96 \pm \sqrt{8.96^2 - 4(-1841.118)}}{2} = 38.6615 \text{ mm}$$

$$\alpha = \beta_1 c = 25.13 \text{ mm}$$

$$f'_s = 600 \left(\frac{38.6615 - 50}{38.6615} \right) = -175.966 \text{ MPa}$$

$$M_{no} = (\alpha_1 f'_c)(\beta_1 c)b \left(\frac{h}{2} - \frac{a}{2} \right) + A'_s f'_s \left(\frac{h}{2} - d' \right) - A_s f_y \left(d - \frac{h}{2} \right)$$

$$M_{no} = (\alpha_1 f'_c)(\beta_1 c)b \left(\frac{h}{2} - \frac{a}{2} \right) + A'_s f'_s \left(\frac{h}{2} - d' \right) - A_s f_y \left(d - \frac{h}{2} \right)$$

$$= [(0.85 \times 58)(25.13)(175) \left(\frac{175}{2} - \frac{25.13}{2} \right) + 339.3(-175.966) \left(\frac{175}{2} - 50 \right) - 339.3(454) \left(125 - \frac{175}{2} \right)] / 10^6 = 19.555 \text{ kN.m}$$

$$P_{no} = 1736.6 \text{ kN}$$

$$C_b = 71.1575 \text{ mm}$$

$$a_b = 46.252 \text{ mm}$$

$$f'_s = 178.4 \text{ MPa}$$

$$P_{nb} = 299.9 \text{ kN}$$

$$M_{nb} = 33.3716 \text{ kN.m}$$

$$X_0 = M_n / M_{n0} = 22.14 / 19.555 = 1.1322$$

$$X_b = M_n / M_{nb} = 22.14 / 33.3716 = 0.6634$$

$$Z = (P_n - P_{nb}) / (P_{no} - P_{nb}) = (1476 - 299.9) / (1763.6 - 299.9) = 0.8035$$

Application of equations 10 and 12:

$$Z = 0.92 + 0.044X - 0.064X^2$$

$$AP_n^2 + BP_n + C = 0$$

$$A = -3.8 \text{ E-08}$$

$$B = -7.17 \text{ E-04}$$

$$C = 1.1339$$

$$P_{n \text{ cal.}} = 1467.6$$

$$R = P_{n \text{ cal.}} / P_{n \text{ exp.}} = 1467.6 / 1476 = 0.9943$$

Application of equations 10 and 13:

$$Z = 0.92 + 0.007X - 0.326X^2$$

$$AP_n^2 + BP_n + C = 0$$

$$A = -6.586 \text{ E-}08$$

$$B = -6.8 \text{ E-}04$$

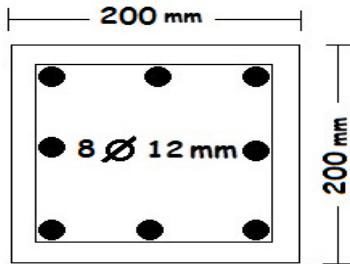
$$C = 1.1249$$

$$P_{n \text{ cal.}} = 1450.493 \text{ kN}$$

$$R = P_{n \text{ cal.}} / P_{n \text{ exp.}} = 1450.493 / 1476 = 0.9827$$

Example 2: Specimen/series J/Column #2/ref [28].

Experimental data:



$$f'_c = 92 \text{ MPa}$$

$$f_y = 560 \text{ MPa}$$

Eccentricity (e) = 10 mm

$$P_{n \text{ exp.}} = 2626 \text{ kN}$$

Solution: The following results are obtained using computer programs:

$$M_{no} = 31.048 \text{ Kn.mm}$$

$$M_{nb} = 64.1132 \text{ Kn.mm}$$

$$P_{no} = 3149.912 \text{ kN}$$

$$P_{nb} = 525.336 \text{ kN}$$

$$X_o = M_n / M_{no} = 0.8458$$

$$X_b = M_n / M_{nb} = 0.4096$$

$$Z = 0.8004$$

Application of equations 10 and 12:

$$AP_n^2 + BP_n + C = 0$$

$$A = -6.688 \text{ E-}09$$

$$B = -3.95 \text{ E-}04$$

$$C = 1.12916$$

$$P_{n \text{ cal.}} = 2731.064 \text{ kN}$$

$$R = P_{n \text{ cal.}} / P_{n \text{ exp.}} = 2731.064 / 2626 = 1.04$$

Application of equations 10 and 13:

$$AP_n^2 + BP_n + C = 0$$

$$A = -7.931 \text{ E-}09$$

$$B = -3.799 \text{ E-}04$$

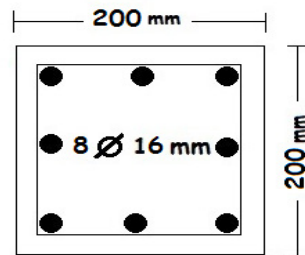
$$C = 1.12016$$

$$P_{n \text{ cal.}} = 2786.328 \text{ kN}$$

$$R = P_{n \text{ cal.}} / P_{n \text{ exp.}} = 2786.328 / 2626 = 1.061$$

Example 3: Specimen/series A/Column #3/ref [28].

Experimental data:



$$f'_c = 103 \text{ MPa}$$

$$F_y = 576 \text{ MPa}$$

Eccentricity (e) = 10 mm

$$P_{n \text{ exp.}} = 3476 \text{ kN}$$

Solution: The following results are obtained using computer programs:

$$M_{no} = 56.153 \text{ kN.mm}$$

$$M_{nb} = 81.3775 \text{ kN.mm}$$

$$P_{no} = 3892 \text{ kN}$$

$$P_{nb} = 472.6866 \text{ kN}$$

$$X_o = M_n / M_{no} = 0.619$$

$$X_b = M_n / M_{nb} = 0.427145$$

$$Z = 0.8783$$

Application of equations 10 and 12:

$$AP_n^2 + BP_n + C = 0$$

$$A = -2.0446 \text{ E-}09$$

$$B = -3.003 \text{ E-}04$$

$$C = 1.06724$$

$$P_{n \text{ cal.}} = 3471.932 \text{ kN}$$

$$R = P_{n \text{ cal.}} / P_{n \text{ exp.}} = 3471.932 / 3476 = 0.9988$$

Application of equations 10 and 13:

$$AP_n^2 + BP_n + C = 0$$

$$A = -4.92276 \text{ E-}09$$

$$B = -2.916 \text{ E-}04$$

$$C = 1.05824$$

$$P_{n \text{ cal.}} = 3430.46 \text{ kN}$$

$$R = P_{n \text{ cal.}} / P_{n \text{ exp.}} = 3430.46 / 3476 = 0.9869$$

Conclusions

1. An alternative method is proposed for analysis and design of normal and high strength concrete square and rectangular tied columns subjected to compression load with uni-axial and bi-axial bending conditions.
2. General 2nd and 4th degree equations are proposed for this purpose. The coefficient values of these equations are

determined using the experimental data of previous studies by applying the principles of least square method.

3. The method is applied on some specimens found in previous studies, and good agreement is found with the experimental results.
4. Computer programs are prepared to find the coefficient of the general equations, and performing all calculations and finding the theoretical load for normal and high strength concrete columns.

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