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The Advanced Conjectures for the Prime Number Theorem

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Abstract

This paper examines the distribution of the prime numbers using epidemiological statistical methods. First, we will show more advanced conjectures of the prime number theorem in two forms.

One is, like conventional conjecture, when drawn on coordinates, it becomes a curve. However, we aim for the curve to pass through the center of the dispersion of the prime counting function. And we obtain the results worthy of publication in this regard.

The other one is what we call the corridors for the prime number theorem. As x increases, a larger proportion of π(x) gathers in what we call the main corridor.

Next, we will clarify the process of acquiring this theory. Initially, we will clarify how the basic form of the conjecture is determined. Then, we will describe the process of obtaining the value for α used in this conjecture. Through this, we can share the results and origins of these conjectures.

At the end, we declare our achievement, even though we acknowledge that this is only a prediction rather than a proof.

Keywords: Prime number theorem • Advanced conjectures • Distribution of the prime numbers

Introduction

This paper attempts to clarify the distribution of the prime numbers using epidemiological statistical methods. First, we will show more advanced conjectures for the prime number theorem in two forms. Next, we will clarify the process of acquiring this theory. Through this, we can share the results and origins of these conjectures.

Although this is not a mathematical proof, it shows sufficient results within the range of prime counting functions that we know as of November 2024. In the future, whenever a larger prime counting function is announced, the numbers presented in this paper will be compared. And our confidence in this prediction will grow as the year goes on.

Now, let's share the advanced conjectures for the prime number theorem.

The Advanced Conjectures for the Prime Number Theorem

This section presents the advanced conjectures for the prime number theorem in two forms.

The first one, like a traditional conjecture, when graphed on coordinates, becomes a curve. However, it is set with the aim of passing through the center of the dispersion of the prime counting function.

The other one is called the corridors for the prime number theorem, which represents the location of the prime counting function by paths. As the number grows, the path we call the main corridor contains a larger proportion of the numbers on the prime counting function.

So let's look at them in turn.

The Advanced Conjecture based on li(x)

Basic concept: Here, we represent the new conjecture as ali(x). "ali(x)" is an abbreviation for the advanced conjecture for the prime number theorem based on li(x).

The basic form of ali(x) is as follows.

$$ali(x) = li\left(\left(1-\left(\frac{1}{\alpha-\frac{1}{\log_{10} x}}\right)^{\log_{10} x}\right)x\right) \sim \pi(x)$$

$$\alpha \sim 3.20405716$$

Comparison with π(x): We shall compare ali(x) and π(x). Here, in creating the Tables, we used the approximate value to the 8th decimal place that we showed above for the value for α of ali(x).

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① The first 30 Numbers from 2 to 31 (Table 1).

Table 1. Table of π(x) and ali(x) of the first 30 numbers. The Table compares the exact values of π(x) to the approximation ali(x).

x	π(x)	ali(x)	ali(x) -π(x)
2	1	1.512320 -3.478255 i	0.512320 -3.478255 i
3	2	-0.053019438260	-2.053019438260
4	-	-1.981508269188	-3.981508269188
5	3	0.456465819067	-2.543534180933
6	-	1.540629817525	-1.459370182475
7	4	2.285938003518	-1.714061996482
8	-	2.895334382513	-1.104665617487
9	-	3.429306908141	-0.57
10	-	3.914468285040	-0.09
11	5	4.365026492442	-0.63
12	-	4.789525093163	-0.21
13	6	5.193536165899	-0.81
14	-	5.580915218987	-0.42
15	-	5.954452892571	-0.05
16	-	6.316241929312	0.32
17	7	6.667897246403	-0.33
18	-	7.010694748871	0.01
19	8	7.345662593020	-0.65
20	-	7.673643303305	-0.33
21	-	7.995337312738	-0.005
22	-	8.311334263394	0.31
23	9	8.622136007402	-0.38
24	-	8.928173837666	-0.07
25	-	9.229821617500	0.23
26	-	9.527405938049	0.53
27	-	9.821214083747	0.82
28	-	10.111500355733	1.11
29	10	10.398491147656	0.40
30	-	10.682389061301	0.68
31	11	10.963376274546	-0.04

② 100 and its Closest Reversals (Table 2).

Table 2. Table of π(x) and ali(x) of 100 and its closest reversals. The Table compares the exact values of π(x) to the approximation ali(x).

x	π(x)	ali(x)	ali(x) -π(x)
31	11	10.963376274546	-0.04
32	11	11.241617320874	0.24
100	25	27.109119012287	2.11
108	28	28.770600958261	0.77
109	29	28.976489688539	-0.02

③ 1,000 and its Closest Reversals (Table 3).

Table 3. Table of π(x) and ali(x) of 1,000 and its closest reversals. The Table compares the exact values of π(x) to the approximation ali(x).

x	π(x)	ali(x)	ali(x) -π(x)
113	30	29.796278571466	-0.20
114	30	30.000308316136	0.0003
1,000	168	171.471469625543	3.47
1,626	257	257.668686622857	0.67
1,627	258	257.802338475763	-0.20

④ 10,000 and its Closest Reversals (Table 4).

Table 4. Table of π(x) and ali(x) of 10,000 and its closest reversals. The Table compares the exact values of π(x) to the approximation ali(x).

x	π(x)	ali(x)	ali(x) -π(x)
9,973	1,229	1,228.952200674199	-0.05
9,974	1,229	1,229.060257118275	0.06
10,000	1,229	1,231.869321259752	2.87
10,332	1,267	1,267.671578162126	0.67
10,333	1,268	1,267.779230304922	-0.22

⑤ 100,000 and its Closest Reversals (Table 5).

Table 5. Table of π(x) and ali(x) of 100,000 and its closest reversals. The Table compares the exact values of π(x) to the approximation ali(x).

x	π(x)	ali(x)	ali(x) -π(x)
99,881	9,584	9,583.979446853340	-0.02
99,882	9,584	9,584.066170280445	0.07
100,000	9,592	9,594.299010318085	2.29
102,022	9,769	9,769.484545593134	0.48
102,023	9,770	9,769.571111020477	-0.436

⑥ 1,000,000 and its Closest Reversals (Table 6).

Table 6. Table of π(x) and ali(x) of 1,000,000 and its closest reversals. The Table compares the exact values of π(x) to the approximation ali(x).

x	π(x)	ali(x)	ali(x) -π(x)
975,908	76,791	76,790.936749822652	-0.06
975,909	76,791	76,791.009220855387	0.009
1,000,000	78,498	78,535.366483611046	37.37
1,021,296	80,074	80,074.821811797455	0.82
1,021,297	80,075	80,074.894045601398	-0.11

⑦ 10,000,000 and its Closest Reversals (Table 7).

Table 7. Table of π(x) and ali(x) of 10,000,000 and its closest reversals. The Table compares the exact values of π(x) to the approximation ali(x).

x	π(x)	ali(x)	ali(x) -π(x)
9,866,927	656,414	656,413.984376879959	-0.02
9,866,928	656,414	656,414.046459843439	0.05
10,000,000	664,579	664,672.113408627484	93.11
10,675,878	706,512	706,512.157614764212	0.16
10,675,879	706,513	706,512.219395851925	-0.78

⑧ From 10^8 to 10^{29} (Table 8).

Table 8. Table of π(x) and ali(x) from 10^8 and 10^{29}. The Table compares the exact values of π(x) to the approximation ali(x).

x	π(x)	ali(x)	ali(x) -π(x)
10^8	5,761,455	5,761,537	82
10^9	50,847,534	50,847,372	-162
10^{10}	455,052,511	455,050,385	-2,126
10^{11}	4,118,054,813	4,118,051,569	-3,244
10^{12}	37,607,912,018	37,607,907,864	-4,154
10^{13}	346,065,536,839	346,065,523,648	-13,191
10^{14}	3,204,941,750,802	3,204,941,711,750	-39,052
10^{15}	29,844,570,422,669	29,844,570,444,511	21,842
10^{16}	279,238,341,033,925	279,238,341,233,157	199,232
10^{17}	2,623,557,157,654,233	2,623,557,156,754,856	-899,377
10^{18}	24,739,954,287,740,860	24,739,954,283,590,369	-4,150,491
10^{19}	234,057,667,276,344,607	234,057,667,299,061,596	22,716,989
10^{20}	2,220,819,602,560,918,840	2,220,819,602,554,912,420	-6,006,420
10^{21}	21,127,269,486,018,731,928	21,127,269,485,936,261,994	-82,469,934
10^{22}	201,467,286,689,315,906,290	201,467,286,689,223,043,217	-92,863,073
10^{23}	1,925,320,391,606,803,968,923	1,925,320,391,608,008,771,865	1,204,802,942
10^{24}	18,435,599,767,349,200,867,866	18,435,599,767,348,267,660,875	-933,206,991
10^{25}	176,846,309,399,143,769,411,680	176,846,309,399,144,763,204,995	993,793,315
10^{26}	1,699,246,750,872,437,141,327,603	1,699,246,750,872,430,489,503,664	-6,651,823,939
10^{27}	16,352,460,426,841,680,446,427,399	16,352,460,426,841,700,631,349,591	20,184,922,192
10^{28}	157,589,269,275,973,410,412,739,598	157,589,269,275,973,368,131,101,133	-42,281,638,465
10^{29}	1,520,698,109,714,272,166,094,258,063	1,520,698,109,714,272,287,746,702,124	121,652,444,061

The Corridors for the Prime Number Theorem

Basic concept: Suppose replacing α in ali(x) with "a" and substitute an arbitrary number to "a". And let's represent it as ali[a](x).

That is, it becomes the following formula:

$$ali[a](x) = li\left(\left(1-\left(\frac{1}{a-\frac{1}{\log_{10} x}}\right)^{\log_{10} x}\right)x\right)$$

Then, let's create two mathematical formulas in which "a" of ali[a](x) is filled with the numbers "m" and "m+0.01", respectively, and then consider drawing them on the coordinate plane. In other words, suppose we draw graphs of two mathematical expressions, ali[m](x) and ali[m+0.01](x), on the coordinate plane. In this case, you can imagine that there is a space between the two graphs.

This space is the corridor. More precisely, the smaller formula itself (in this case ali[m](x)) is also a part of this corridor.

We denote the corridor created in this way as ac[m](x). Here, "m" is a decimal number up to two decimal places. "ac" means "ali[a](x) corridor". For example, ac[3.00](x) is the area that combines the formula of ali[3.00](x) and the space created between ali[3.00](x) and ali[3.01](x). In other words, ac[3.00](x) is the area where "a" is equal or more than 3.00, less than 3.01 in ali[a](x).

Affiliated corridor: Here, we will discuss how to assign a natural number x greater than or equal to 2 to a corridor. The corridor to which it belongs is determined by the prime counting function of that number.

For example, the corridor to which 10 belongs is determined as follows.

The prime counting function 10 is 4. And ali[3.27](10) is 3.99155 and ali[3.28](10) is 4.00276. Therefore, 10 belongs to ac[3.27](x).

The main corridor is ac[3.20](x). This is the corridor that contains ali(x). As x increases, the proportion of natural numbers that belong to this main corridor tends to increase. This point is very different from ali(x). In ali(x), as x increases, the numerical error, whether positive or negative, tends to increase. However, in the case of ac[3.20](x), as x becomes larger, broaden the corridor becomes, so the affiliation of the natural numbers converge to it.

Statistics: Here, we will statistically show how the affiliation of natural numbers converges to ac[3.20](x).

For the numbers up to the 7th power of 10, we will analyze the affiliated corridors of 100 consecutive natural numbers. What is depicted here is how the affiliations of 100 consecutive natural numbers, which are scattered at first, gradually come together.

For numbers from 10 to the 8th power to 10 to the 29th power, we showed the affiliated corridor for each number. What is depicted here is how the affiliation of these numbers converges to ac[3.20](x).

So let's take a look at the statistics.

We start statistics from 9. It is because the numbers from 2 to 8 are not appropriate for this statistics, as shown in the following Table (Table 9).

Table 9. The affiliated Corridors of the first 7 numbers Table shows the affiliated corridors of the first 7 numbers which is inappropriate for statistics.

x	Affiliated Corridor
2	ac[1,032,288.63](x)
3	ac[388.90](x)
4	ac[9.31](x)
5	ac[12.11](x)
6	ac[5.51](x)
7	ac[7.95](x)
8	ac[4.91](x)

Affiliated corridors of the 100 numbers from 9 to 108: Following Table shows the affiliated corridors of 100 numbers from 9 to 108. The numbers are scattered across 56 different affiliations. The average value of "a", which is in the nest of ac[a](x) is 2.9063.

The closest number on the main corridor to 100 in each side is 31 and 110, which comes after 109 of ac[3.21](x) (Table 10).

Table 10. The affiliated corridors of the 100 numbers from 9 to 108.

A	B
ac[2.54](x)	2
ac[2.58](x)	2
ac[2.60](x)	4
ac[2.62](x)	1
ac[2.63](x)	2
ac[2.64](x)	2
ac[2.65](x)	1
ac[2.66](x)	1
ac[2.67](x)	2
ac[2.68](x)	3
ac[2.69](x)	2
ac[2.70](x)	1
ac[2.71](x)	3
ac[2.72](x)	2
ac[2.73](x)	2
ac[2.74](x)	1
ac[2.75](x)	2
ac[2.76](x)	4
ac[2.77](x	4
ac[2.78](x)	1
ac[2.79](x)	4
ac[2.80](x)	1
ac[2.81](x)	2
ac[2.82](x)	2
ac[2.83](x)	2
ac[2.85](x)	1
ac[2.86](x)	2
ac[2.87](x)	4
ac[2.88](x)	3
ac[2.90](x)	4
ac[2.92](x)	2
ac[2.93](x)	1
ac[2.95](x)	3
ac[2.97](x)	1
ac[2.98](x)	3
ac[2.99](x)	2
ac[3.00](x)	1
ac[3.04](x)	1
ac[3.05](x)	1
ac[3.07](x)	1
ac[3.11](x)	2
ac[3.19](x)	1
ac[3.20](x)	1
ac[3.22](x)	1
ac[3.24](x)	1
ac[3.25](x)	1
ac[3.27](x)	1
ac[3.39](x)	1
ac[3.49](x)	1
ac[3.51](x)	1
ac[3.53](x)	1
ac[3.62](x)	1
ac[3.82](x)	1
ac[3.91](x)	1
ac[3.94](x)	1
ac[4.25](x)	1

A: Corridor for the prime number theorem
B: Number of the numbers affiliated the corridor

Affiliated corridors of the 100 numbers from 901 to 1,000: Following Table shows the affiliated corridors of 100 numbers from 901 to 1,000. The numbers belong to 16 different corridors. The average value of "a", which is in the nest of ac[a](x) is 2.7898.

The main reason for the value being much smaller than the main corridor is the existence of a prime number dessert from 888 to 906. The affiliated corridor of 887 just before the dessert is ac[3.02](x).

The closest number on the main corridor to 1,000 in each side is 114 and 1,621 (Table 11).

Table 11. The affiliated corridors of the 100 numbers from 901 to 1,000.

A	B
ac[2.71](x)	1
ac[2.72](x)	2
ac[2.73](x)	2
ac[2.74](x)	4
ac[2.75](x)	6
ac[2.76](x)	9
ac[2.77](x)	10
ac[2.78](x)	12
ac[2.79](x)	9
ac[2.80](x)	9
ac[2.81](x)	13
ac[2.82](x)	8
ac[2.83](x)	7
ac[2.84](x)	2
ac[2.85](x)	4
ac[2.86](x)	2

A: Corridor for the prime number theorem
B: Number of the numbers affiliated the corridor

Affiliated corridors of the 100 numbers from 9,901 to 10,000: Following Table shows the affiliated corridors of 100 numbers from 9,901 to 10,000. The numbers belong to 17 different corridors. The average value of "a", which is in the nest of ac[a](x) is 3.1679.

The closest number on the main corridor to 10,000 in each side is 9,974 and 10,334, which comes after 10,333 of ac[3.21](x) (Table 12).

Table 12. The affiliated corridors of the 100 numbers from 9,901 to 10,000.

A	B
ac[3.07](x)	2
ac[3.08](x)	3
ac[3.09](x)	2
ac[3.10](x)	2
ac[3.11](x)	2
ac[3.12](x)	2
ac[3.13](x)	4
ac[3.14](x)	5
ac[3.15](x)	8
ac[3.16](x)	11
ac[3.17](x)	11
ac[3.18](x)	13
ac[3.19](x)	10
ac[3.20](x)	10
ac[3.21](x)	6
ac[3.22](x)	5
ac[3.23](x)	4

A: Corridor for the prime number theorem
B: Number of the numbers affiliated the corridor

Affiliated corridors of the 100 numbers from 99,901 to 100,000: Following Table shows the affiliated corridors of 100 numbers from 99,901 to 100,000. The numbers belong to 6 different corridors. The average value of "a", which is in the nest of ac[a](x) is 3.1723.

The closest number on the main corridor to 100,000 in each side is 99,930 and 102,019 (Table 13).

Table 13. The affiliated corridors of 100 numbers from 99,901 to 100,000.

A	B
ac[3.15](x)	9
ac[3.16](x)	27
ac[3.17](x)	21
ac[3.18](x)	21
ac[3.19](x)	19
ac[3.20](x)	3

A: Corridor for the prime number theorem
B: Number of the numbers affiliated the corridor

Affiliated corridors of the 100 numbers from 999,901 to 1,000,000: Following Table shows the affiliated corridors of 100 numbers from 999,901 to 1,000,000. The numbers' affiliation narrows down to 2 different corridors. The average value of "a", which is in the nest of ac[a](x) is 3.0305.

The closest number on the main corridor to 1,000,000 in each side is 976,650 and 1,021,291 (Table 14).

Table 14. The affiliated corridors of 100 numbers from 999,901 to 1,000,000

A	B
ac[3.03](x)	95
ac[3.04](x)	5

A: Corridor for the prime number theorem
B: Number of the numbers affiliated the corridor

Affiliated corridors of the 100 numbers from 9,999,901 to 10,000,000: Following Table shows the affiliated corridors of 100 numbers from 9,999,901 to 10,000,000. All numbers belong to ac[3.06](x).

The closest number on the main corridor to 10,000,000 in each side is 9,867,286 and 10,673,893 (Table 15).

Table 15. The affiliated corridors of 100 numbers from 9,999,901 to 10,000,000.

A	B
ac[3.06](x)	100

A: Corridor for the prime number theorem
B: Number of the numbers affiliated the corridor

Affiliated corridors of the numbers power of ten from 10^8 to 10^{29}: Following Table shows the affiliated corridors of the numbers from 10^8 to 10^{29}. The average value of "a", which is in the nest of ac[a](x) is 3.2136. What is even more important is the trend that the affiliation of the numbers converges to the main corridor (Table 16).

Table 16. The affiliated corridors of the numbers power of ten from 10^8 to 10^{29}.

A	B
10^{8}	ac[3.15](x)
10^{9}	ac[3.23](x)
10^{10}	ac[3.37](x)
10^{11}	ac[3.27](x)
10^{12}	ac[3.23](x)
10^{13}	ac[3.23](x)
10^{14}	ac[3.23](x)
10^{15}	ac[3.19](x)
10^{16}	ac[3.19](x)
10^{17}	ac[3.22](x)
10^{18}	ac[3.23](x)
10^{19}	ac[3.16](x)
10^{20}	ac[3.20](x)
10^{21}	ac[3.22](x)
10^{22}	ac[3.21](x)
10^{23}	ac[3.17](x)
10^{24}	ac[3.21](x)
10^{25}	ac[3.20](x)
10^{26}	ac[3.20](x)
10^{27}	ac[3.19](x)
10^{28}	ac[3.20](x)
10^{29}	ac[3.20](x)

A: x
B: Affiliated Corridor

The Process of Obtaining the Conjecture

Here, we will explain the process of obtaining ali(x), dividing it into establishing its basic form and refining the value for α.

Establishment of the basic form of ali(x)

As already shown, the basic form of ali(x) is as follows.

$$ali(x) = li\left(\left(1-\left(\frac{1}{\alpha-\frac{1}{\log_{10}x}}\right)^{\log_{10}x}\right)x\right) \sim \pi(x)$$

In this subsection, we will describe the process of establishing this basic form.

Until now, we have always used "li(x) -π(x)" when dealing with error in li(x). However, here we first find x' such that li(x') equals π(x). Next, check the error between x' and x.

Regarding the error, the results are initially expressed in the form "x' = β × x". For example, 10^3 and 10^4 are as follows, respectively:

For 10^3 $\pi(10^3) = 168$

$li(0.93394399493 \times 10^3) \sim 168.000000001131$

For 10^4 $\pi(10^4) = 1{,}229$

$li(0.984229629389 \times 10^4) \sim 1{,}229.000000001064$

In this form, as shown in Table 17, we cannot find any regularity or law in β.

Next, if x is a number that is a power of 10, try expressing it in the following format. Here we show the case where x is 10^n.

$$x' = \left(1-\left(\frac{1}{\alpha-\frac{1}{n}}\right)^{n}\right)x$$

In other expressions, it is as follows.

$$li(x') = li\left(\left(1-\left(\frac{1}{\alpha-\frac{1}{n}}\right)^{n}\right)x\right) = \pi(x)$$

For example, 10^3 and 10^4 are as follows, respectively:

For 10^3

$$li(x') \sim li\left(\left(1-\left(\frac{1}{2.80712182617-\frac{1}{3}}\right)^{3}\right)\times 10^3\right) \sim 168.000000000041$$

For 10^4

$$li(x') \sim li\left(\left(1-\left(\frac{1}{3.07188639103-\frac{1}{4}}\right)^{4}\right)\times 10^4\right) \sim 1{,}229.000000000121$$

When expressed in this format, a law is born in the value for α. In other words, as shown in the Table below, the value for α converges around 3.2.

When we convert this formula into a general number format, it is as follows.

$$li\left(x^{'}\right)=li\left(\left(1-\left(\frac{1}{\alpha-\frac{1}{\log_{10}x}}\right)^{\log_{10}x}\right)x\right)=\pi\left(x\right)$$

This li(x') is the prototype of ali(x). In this way, the basic form of ali(x) is derived (Table 17).

Table 17. Table of β of li(β × x) which equals π(x) and α of li((1-(1/(α-1/log(10,x)))^log(10,x))x) which equals such li(β × x).

x	π(x)	β	α
10	4	0.560927669306	3.2775290769
10^2	25	0.7703927733299	2.58692683503
10^3	168	0.93394399493	2.80712182617
10^4	1,229	0.984229629389	3.07188639103
10^5	9,592	0.99564790189	3.16660868879
10^6	78,498	0.998210328217	3.03659532824
10^7	664,579	0.99945295298156	3.06698580548
10^8	5,761,455	0.99986103943127451	3.159859235116081
10^9	50,847,534	0.99996475064585370 3251	3.235470483128315
10^{10}	455,052,511	0.99999285372881584 6654	3.370333557307938
10^{11}	4,118,054,813	0.99999706503684359406338021	3.2747049480987775
10^{12}	37,607,912,018	0.9999989427596510 33503	3.2309767172449604
10^{13}	346,065,536,839	0.99999967381035088656234	3.2316649674835933
10^{14}	3,204,941,750,802	0.9999998984914722 01708	3.2303260677831371
10^{15}	29,844,570,422,669	0.9999999636438422186 05633	3.1996744083711893378
10^{16}	279,238,341,033,925	0.9999999881568588090665092309	3.19151985051739102 1103
10^{17}	2,623,557,157,654,233	0.99999999688547713917473993 2015	3.223932989292175652843
10^{18}	24,739,954,287,740,860	0.99999999909026707255495854899161	3.234497233997829043089
10^{19}	234,057,667,276,344,607	0.99999999995630436730589084338 2116	3.1615442178296429772822
10^{20}	2,220,819,602,560,918,840	0.999999999897422300844394 1129	3.20825616323851162233
10^{21}	21,127,269,486,018,731,928	0.999999999971113426801989160 4838	3.2235541388443170 1393379
10^{22}	201,467,286,689,315,906,290	0.99999999999021129294735595591729	3.2108033639545883285088
10^{23}	1,925,320,391,606,803,968,923	0.99999999999616034073856630449596 93	3.1791825541080450635321 19
10^{24}	18,435,599,767,349,200,867,866	0.99999999999905242688584909154049764	3.2110478091840185281541
10^{25}	176,846,309,399,143,769,411,680	0.99999999999968246786893514587198 2743	3.201757030101441242795
10^{26}	1,699,246,750,872,437,141,327,603	0.999999999999990667199791768275956 94690	3.2091486393917024400 10064794
10^{27}	16,352,460,426,841,680,446,427,399	0.999999999999996837629687123062256 76219888	3.19931126567575035593 9559886
10^{28}	157,589,269,275,973,410,412,739,598	0.999999999999999079498355234215532108339220	3.2073612263939187117293 0178811
10^{29}	1,520,698,109,714,272,166,094,258,063	0.99999999999999969609419287793696117177954	3.20109733779278595453347 23558

fining the value for α

Once the basic form of ali(x) is determined, the next biggest challenge is what value to use for α. In other words, we need to find the true value of α for the prime number theorem hold.

By referring to Table 17 and repeating trial and error, three values become the final tentative candidates for the true value for α. Let's start by comparing these three values.

Comparison of the three tentative candidates: The three final tentative candidates for the value for α are 3.2, 3.205, and 3.21. Here, we substitute 3.2, 3.205, and 3.21 for α in the formula for ali(x), and denote the resulting values ali[3.2](x), ali[3.205](x), and ali[3.21](x). This is almost identical in format and meaning to the one described in the corridors for the prime number theorem.

However, it differs from the corridors in the following two points. One is that there is no limit to the number of decimal digits in α. The other is that α can be set to any arbitrary number only at the research stage. This is because α is a symbol with a true value, just like π for pi and e for Napier's number.

Now, we compare ali[3.2](x), ali[3.205](x), and ali[3.21](x), and the results are shown in the Table below. This Table shows that the true value for α is greater than 3.2, less than 3.21, and of those three numbers, it is closest to 3.205 (Table 18).

Table 18. Table of the three approximations on error. The Table compares the three approximations ali[3.2](x), ali[3.205](x), and ali[3.21](x) by the exact value of π(x).

x	ali[3.2](x) -π(x)	ali[3.205](x) -π(x)	ali[3.21](x) -π(x)
10	-0.090	-0.084	-0.078
10^2	2.10	2.11	2.12
10^3	3.45	3.48	3.51
10^4	2.8	2.9	3.0
10^5	2.1	2.4	2.6
10^6	36.6	37.5	38.4
10^7	91	94	96
10^8	75	84	93
10^9	-184	-157	-130
10^{10}	-2,195	-2,110	-2,027
10^{11}	-3,458	-3,195	-2,936
10^{12}	-4,821	-4,001	-3,197
10^{13}	-15,270	-12,713	-10,213
10^{14}	-45,532	-37,564	-29,784
10^{15}	1,640	26,478	50,691
10^{16}	136,235	213,675	289,048
10^{17}	-1,095,852	-854,368	-619,693
10^{18}	-4,763,351	-4,010,205	-3,279,438
10^{19}	20,805,064	23,154,292	25,430,167
10^{20}	-11,971,651	-4,643,096	2,445,574
10^{21}	-101,083,257	-78,219,276	-56,138,142
10^{22}	-150,946,925	-79,609,050	-10,821,033
10^{23}	1,023,536,605	1,246,133,327	1,460,439,371
10^{24}	-1,498,931,961	-804,317,483	-136,615,806
10^{25}	-771,906,185	1,395,758,887	3,476,191,993
10^{26}	-12,163,057,326	-5,398,161,426	1,084,384,426
10^{27}	2,982,124,493	24,095,047,000	44,295,248,570
10^{28}	-95,980,573,068	-30,085,654,673	32,862,371,186
10^{29}	-45,975,675,664	159,693,941,703	355,859,674,219

Analysis by IDV-tt(x)

In further analyzing the value for α, we adopted IDV-tt(x). This is because we needed some measure to further narrow down the range of the true value for α.

Basic concept: IDV-tt is an abbreviation for intuitive distancing value at ten times.

There are two types of IDV—tt(x): IDV—tt[n](x) and IDV—tt[p](x). IDV-tt[n](x) stands for intuitive distancing value at ten times by natural numbers. IDV-tt[p](x) stands for intuitive distancing value at ten times by prime numbers. IDV-tt[pp](x) is derived from the IDV-tt[p](x), but this will be discussed later.

IDV-tt(x) is a method for measuring whether the values of two formulas f(x) and g(x) intuitively approach or move away from each other when x is multiplied by 10.

The formula for each is as follows.

$$IDV-tt[n](x)=\frac{\left(\dfrac{g(x)-f(x)}{g\left(\dfrac{x}{10}\right)-f\left(\dfrac{x}{10}\right)}\right)}{10}$$

$$IDV-tt[p](x)=\frac{\left(\dfrac{g(x)-f(x)}{g\left(\dfrac{x}{10}\right)-f\left(\dfrac{x}{10}\right)}\right)}{\left(\dfrac{\pi(x)}{\pi\left(\dfrac{x}{10}\right)}\right)}$$

The basis of whether the two formulas f(x) and g(x) are intuitively approaching or intuitively moving away is whether the value of the IDV-tt(x) is greater than 1 or not. If the value is greater than 1, the two formulas f(x) and g(x) intuitively moving away during x increases by a factor of 10 from x-tenth to x. Conversely, if the value is less than 1, the two formulas are intuitively approaching each other.

As for the basis of this feeling, it must sound strange. In particular, "intuitively approaching" seems like nonsense. So, we are going to digress a bit, but we will repeat the same content in a different way.

For example, suppose f(x) and g(x) are parallel lines. In that case, the value of "f(x) -g(x)" by the IDV-tt[n](x) is 0.1 and is always constant. On the other hand, its value by the IDV-tt[p](x) converges to 0.1 at the limit value of x. Therefore, if the values of the IDV-tt[n](x) for two expressions are greater than 0.1, then they are in moving away, at least numerically.

However, even if the value of the IDV-tt[n](x) is larger than 0.1 and the two formulas are numerically moving away, when the value is smaller than 1, as x increases, they often have an intersection somewhere. For example, in Table 19 below, the values of "ali(x) -π(x)" shown in Table 8 are analyzed by the IDV-tt(x) and displayed from 10 to the 10th power to 10 to the 15th power. Although it's not a proper example, it can be helpful in understanding the feeling of "intuitively approaching" (Table 19).

Table 19. Table of the IDV-tt(x) by "ali(x) -π(x)" from 10^{10} to10^{15}. The Table shows two kind of the IDV-tt(x) to analyze "ali(x) -π(x)" from 10^{10} to10^{15}.

x	ali(x) -π(x)	IDV-tt[n](x)	IDV-tt[p](x)
10^9	-162	--------------	--------------
10^{10}	-2,126	1.312345679012	1.466414093325
10^{11}	-3,244	0.152587017874	0.168611416755
10^{12}	-4,154	0.128051787916	0.140216314399
10^{13}	-13,191	0.317549350024	0.345089780571
10^{14}	-39,052	0.296050337351	0.319671391535
10^{15}	21,842	-0.055930554133	-0.060062572705

The Table above shows an example of two formulas whose values of the IDV-tt(x) are more than 0.1 and which are moving away numerically, but then reach a reversal point. We may understand the intuitive feeling that when the value of the IDV-tt(x) is less than 1, even if it is more than 0.1, the two formulas are likely to have an intersection.

"Intuitively approaching" is a feeling that arises when you are exposed to many such phenomena.

On the other hand, we also noted that this is not a proper example. The reason is that the up-down movement of "ali(x) -π(x)" is too much fast for the IDV-tt(x). In other words, the meandering of "ali(x) -π(x)" is too much small for the IDV-tt(x). In short, there are countless reversal points between ali(x) and π(x) between 10 to the 9th power and 10 to the 15th power.

If the first reversal point were around 10 to the 15th power, numerical sequence of the IDV-tt(x) would have been more beautiful. In other words, in that case, the value of the IDV-tt(x) would have gradually decreased until it became a negative number at the reversal point.

In this way, when the value of the IDV-tt(x) is less than 1, especially when the value follows a downward trend, as x increases, there is a fairly high probability that the two formulas have an intersection point. This is the phenomenon that creates the feeling of "intuitively approaching."

There is one more point that we should add here, although it is completely different from the discussion we have been discussing so far. What we have described above is one fact, but there is also another aspect as well. That is, when the value of the IDV-tt(x) is less than 1, the overwhelming majority of them converge to 1. Let's call this kind of relationship between two formulas "coproportionalize" or "coproportionalization."

This term is made from the term coprime. For example, although neither 14 nor 15 are prime numbers, their relationship is called coprime. So we thought about it accordingly.

For example, when "f(x): $\frac{1}{10}$ x + 10, g(x): $\frac{1}{20}$ x + 10", neither f(x) nor g(x) are proportional expressions. However, "f(x) -g(x)" becomes a proportional expression, and the values by the IDV-tt[n](x) of them are always 1 and constant. Therefore, we defined the relationship between f(x) and g(x) as "coproportional."

Furthermore, when "h(x): $\frac{1}{20}$ x + $\frac{1}{2}$", the value by the IDV-tt[n](x) of "f(x) -h(x)" converges to 1 as x increases. We defined this relationship between f(x) and h(x) as "coproportionalize" or "coproportionalization."At least for the two formulas that are worth analyzing with the IDV-tt[n](x), the overwhelming majority of their relationships are coproportionalization. For example, it has been said that since li(x) and "x/log x" are from the same concept, the two formulas converge at the limit value of x. However, when analyzed using the IDV-tt(x), the relationship between these two formulas is also one of the coproportionalization, as shown in the following Table. (Tables 20 and 20A)

Table 20. Table of the IDV-tt(x) by "li(x) - x/log x" from 10^2 to10^{29}. The Table shows two kind of the IDV-tt(x) to analyze "li(x) - x/log x" from 10^2 to 10^{29}.

x	li(x) - x/log x	IDV-tt[n](x)	IDV-tt[p](x)
10	1.8	---------	---------
10^2	8.4	0.461492654350	0.738388246959
10^3	32.8	0.390479140197	0.581070149102
10^4	160	0.488359987785	0.667571016663
10^5	944	0.588474696182	0.753998542126
10^6	6,245	0.661617480668	0.808458161299
10^7	44,498	0.712518031863	0.841604090186
10^8	333,528	0.749540374263	0.864588532563
10^9	2,594,293	0.777832682114	0.881350115333
10^{10}	20,761,133	0.800261824634	0.894211972319
10^{11}	169,934,747	0.818523485917	0.904483268175
10^{12}	1,416,743,456	0.833698511290	0.912897308780
10^{13}	11,992,967,423	0.846516521823	0.919933234765
10^{14}	02,838,623,526	0.857491060379	0.925907948469
10^{15}	891,606,015,071	0.866995282999	0.931046867447
10^{16}	7,804,293,059,024	0.875307358532	0.935515230697
10^{17}	68,883,742,650,517	0.882639108111	0.939437052330
10^{18}	612,483,092,843,091	0.889154783518	0.942907318837
10^{19}	5,481,624,269,247,736	0.894983769071	0.946000094453
10^{20}	49,347,193,267,404,345	0.900229399965	0.948774016253
10^{21}	446,579,872,175,562,961	0.904975222716	0.951276129539
10^{22}	4,060,704,007,951,976,202	0.909289527128	0.953544627325
10^{23}	37,083,513,773,828,817,525	0.913228684022	0.955610847414
10^{24}	339,996,354,730,854,956,347	0.916839641476	0.957500748467
10^{25}	3,128,516,637,898,199,332,168	0.920161817727	0.959236019709
10^{26}	28,883,358,937,009,080,501,382	0.923228554617	0.960834926017
10^{27}	267,479,615,610,639,940,821,371	0.926068246404	0.962312959465
10^{28}	2,484,097,167,670,613,997,282,501	0.928705225630	0.963683347228
10^{29}	23,130,930,737,546,277,111,573,910	0.931160465001	0.964957451586

Table 20A. Table of the IDV-tt[n](x) by "li(x) - x/log x" from 10^{30} to$10^{10000000}$. The Table shows the IDV-tt[n](x) to analyze "li(x) - x/log x" from 10^{30} to$10^{10000000}$.

x	li(x) - x/log x	IDV-tt[n](x)
10^{30}	215,916,167,612,053,384,041,457,039	0.933452138446
10^{31}	2,020,103,185,741,632,771,851,756,629	0.935596073274
10^{32}	18,940,611,046,104,043,507,744,451,899	0.937606117341
10^{33}	177,945,987,551,364,290,590,150,052,106	0.939494439320
10^{34}	1,674,955,357,143,699,267,726,067,244,126	0.941271775887
10^{35}	15,793,951,951,882,684,695,318,945,656,733	0.942947636456
10^{40}	$1.205293014015345915 \times 10^{36}$	0.950071061011
10^{50}	$7.679067944280513576 \times 10^{45}$	0.960047014249
10^{60}	$5.316755644899477272 \times 10^{55}$	0.966700001858
10^{80}	$2.979587781200565618 \times 10^{75}$	0.975019219051
10^{100}	$1.902716810818488259 \times 10^{95}$	0.980012475113
10^{1000}	$1.887757368679178900 \times 10^{993}$	0.998000130768
10^{10000}	$1.886280817502940492 \times 10^{9991}$	0.999800001313470
10^{100000}	$1.886133352933436144 \times 10^{99989}$	0.999980000013140463
$10^{1000000}$	$1.886118608378658449 \times 10^{999987}$	0.999998000000131410396
$10^{10000000}$	$1.886117133942199106 \times 10^{9999985}$	0.999999800000001314109722

We will not go into further detail on this point in this paper. There is also discussion about whether computer calculations are perfect, and if we push this point too far, things will get out of hand.

Now, let's take the discussion back a little.

Similar to parallel lines, li(x) and ali[α](x) never intersect as x increases, even if the value of the IDV-tt(x) is less than 1. Although keeping an "intuitively approaching," they never actually come into contact with each other. There is something similar to a weightless state, where you feel a continuous falling but never hit the ground. In that respect, the relationships between li(x) and ali[α](x) are very exceptional phenomena for the IDV-tt(x).

The reason why the phrase "intuitive approach" sounds so strange is that we are exposed to exceptional events first.

Now, let's get back to the topic of discussion.

Analysis by the IDV-tt[n](x): Let's assume that f(x) and g(x) are π(x) and li(x), respectively. Now suppose we multiply x by 10, from 100 to 1,000. This gives result in a larger numerical error. That is, the error is 5.126141584080 when x is 100, and 9.609657990152 when x is 1,000. In terms of magnification, that is 1.874637645592 times.

However, the value of the IDV-tt[n](x) comes 0.187463764559. This is because even though the domain has increased by a factor of 10 at the natural number level, the error has only increased by a factor of 0.187463764559. In other words, as x increases from 100 to 1,000, the two formulas are considered to become closer intuitively.

According to this scale, the relationship between li(x) and the three types of ali[α](x) is as shown in the following Table (Table 21).

Table 21. Table of the IDV-tt[n](x) by the three approximations to li(x). The Table compares the IDV-tt[n](x) by the three approximations ali[3.2](x), ali[3.205](x), and ali[3.21](x) to li(x).

x	IDV-tt[n](x) by li(x) -ali[3.2](x)	IDV-tt[n](x) by li(x) -ali[3.205](x)	IDV-tt[n](x) by li(x) -ali[3.21](x)
10	----------	----------	----------
10^{2}	0.134138628739	0.133995530662	0.133852486698
10^{3}	0.203696694847	0.203395036546	0.203094232105
10^{4}	0.232734315663	0.232377520392	0.232021801304
10^{5}	0.249193458701	0.248807738665	0.248423202924
10^{6}	0.259924162047	0.259520412950	0.259117911986
10^{7}	0.267515601107	0.267099413390	0.266684516152
10^{8}	0.273182768173	0.272757420226	0.272333393151
10^{9}	0.277579120103	0.277146719426	0.276715662657
10^{10}	0.281090297479	0.280652289332	0.280215643237
10^{11}	0.283959590315	0.283517013305	0.283075813043
10^{12}	0.286348407282	0.285902034577	0.285457050783
10^{13}	0.288368158723	0.287918582050	0.287470404524
10^{14}	0.290098259826	0.289645942358	0.289195032777
10^{15}	0.291596858233	0.291142169401	0.290688896003
10^{16}	0.292907510471	0.292450749694	0.291995410934
10^{17}	0.294063489856	0.293604903191	0.293147744340
10^{18}	0.295090652559	0.294630444681	0.294171669755
10^{19}	0.296009395485	0.295547738470	0.295087518997
10^{20}	0.296836024969	0.296373064874	0.295911546445
10^{21}	0.297583733329	0.297119595186	0.296656902435
10^{22}	0.298263308385	0.297798100053	0.297334340492
10^{23}	0.298883657426	0.298417472595	0.297952739619
10^{24}	0.299452199882	0.298985120454	0.298519495705
10^{25}	0.299975165541	0.299507263530	0.299040818790
10^{26}	0.300457823809	0.299989162866	0.299521961587
10^{27}	0.300904661905	0.300435298569	0.299967397111
10^{28}	0.301319524814	0.300849509533	0.300380958182
10^{29}	0.301705726215	0.301235104188	0.300765948002
10^{30}	0.302066137189	0.301594949074	0.301125228581
10^{40}	0.304677861891	0.304202575574	0.303728769759
10^{50}	0.306243844797	0.305766104530	0.305289852460
10^{60}	0.307287398284	0.306808024100	0.306330143232
10^{80}	0.308591353272	0.308109938984	0.307630024394
10^{100}	0.309373467712	0.308890830577	0.308409696961
10^{1000}	0.312187484735	0.311700452825	0.311214938148
10^{10000}	0.312468749847	0.311981279099	0.311495326952

Analysis by the IDV-tt[p](x): For example, let us again assume that f(x) and g(x) are π(x) and li(x), respectively. Now suppose we multiply x by 10, just like before, from 100 to 1,000. In this case, in the IDV-tt[p](x), the error of li(x) relative to π(x) is considered to be decreasing, just like in the IDV-tt[n](x).

Because, on the one hand, as x increases from 100 to 1,000, on the other hand at the prime number level, the range expands by 6.72 times, from π(100) to π(1,000), that is, from 25 to 168. In this situation, although the error is increasing at the numerical level, at the magnification level it remains at 1.874637645592 times. Therefore, the value of the IDV-tt[p](x) comes 0.278963935356, which is from 1.874637645592 divided by 6.72.

According to this scale, the relationship between li(x) and the three types of ali[α](x) is as shown in the following Table (Table 22).

Table 22. Table of the IDV-tt[p](x) by three approximations to li(x). The Table compares the IDV-tt[p](x) by the three approximations ali[3.2](x), ali[3.205](x), and ali[3.21](x) to li(x).

x	IDV-tt[p](x) by li(x) -ali[3.2](x)	IDV-tt[p](x) by li(x) -ali[3.205](x)	IDV-tt[p](x) by li(x) -ali[3.21](x)
10	----------	----------	----------
10^2	0.214621805982	0.214392849059	0.214163978716
10^3	0.303120081618	0.302671185336	0.302223559680
10^4	0.318139666651	0.317651939999	0.317165684451
10^5	0.319285613786	0.318791399937	0.318298703496
10^6	0.317612240102	0.317118882140	0.316627049323
10^7	0.315981089617	0.315489501658	0.314999437974
10^8	0.315114030900	0.314623395612	0.314134283939
10^9	0.314520584108	0.314030637625	0.313542213905
10^{10}	0.314090090982	0.313600661001	0.313112752976
10^{11}	0.313780487300	0.313291431693	0.312803897418
10^{12}	0.313550626325	0.313061849585	0.312574593697
10^{13}	0.313377761944	0.312889194370	0.312402147289
10^{14}	0.313244413873	0.312756007221	0.312269120785
10^{15}	0.313139352357	0.312651072178	0.312164312000
10^{16}	0.313055105227	0.312566926235	0.312080267074
10^{17}	0.312986514612	0.312498417835	0.312011840750
10^{18}	0.312929920838	0.312441891773	0.311955382289
10^{19}	0.312882675379	0.312394702753	0.311908249613
10^{20}	0.312842823828	0.312354898741	0.311868493061
10^{21}	0.312808897912	0.312321013244	0.311834647917
10^{22}	0.312779776687	0.312291926676	0.311805595948
10^{23}	0.312754592742	0.312266772671	0.311780471834
10^{24}	0.312732666157	0.312244872129	0.311758597293
10^{25}	0.312713457852	0.312225686621	0.311739434544
10^{26}	0.312696535941	0.312208784776	0.311722552734
10^{27}	0.312681551105	0.312193817698	0.311707603384
10^{28}	0.312668218338	0.312180500720	0.311694302170
10^{29}	0.312656303226	0.312168599711	0.311682415240

The three tentative candidates on the IDV-tt: First, let us discuss the interpretation of the results in Tables 21 and 22.

On the one hand, the value of the IDV-tt[n](x) in Table 21 continues to increase as x increases. There are no exceptions, at least not within the range of numbers shown in this Table.

On the other hand, the value of the IDV-tt[p](x) in Table 22 peaks at 10 to the fifth power, and then continues to get smaller as x gets larger. Again, there are no exceptions in this regard within the range of numbers shown in this Table, from 10 to the fifth power onwards. As long as x is within a numerical range where the exact value of π(x) is known, the IDV-tt[p](x) provides a more stable value than the IDV-tt[n](x).

When the values for α in the following two "li(x) -ali[α](x)" are the same, the value of the IDV-tt[n](x) by "li(x) -ali[α](x)" and the value of the IDV-tt[p](x) by "li(x) -ali[α](x)" converge as x increases. This is a natural result of the fact that the value of (π(x)/π(x/10)) converges to 10 as x becomes larger.

Therefore, the following important inferences can be made from the results in Tables 21 and 22.

First, when the value for α is 3.2, the limit values of the IDV-tt[n](x) and it of the IDV-tt[p](x) converge somewhere between 0.312468749847 and 0.312656303226. The important thing here is that this converged value is larger than the converged value when α is substituted with its true value. It is because 3.2 is smaller than the true value for α.

Next, when the value for α is 3.21, the limit values of the IDV-tt[n](x) and it of the IDV-tt[p](x) converge somewhere between 0.311495326952 and 0.311682415240. And this converged value is smaller than the converged value when α is substituted with its true value. It is because 3.21 is larger than the true value for α.

And, when the value for α is 3.205, the limit values of the IDV-tt[n](x) and it of the IDV-tt[p](x) converge somewhere between 0.311981279099 and 0.312168599711. This converged value is around the converged value when α is substituted with its true value. It is because 3.205 is around the true value for α.

This leads to the following hypothesis.

Hypothesis regarding the true value for α: As a result of the analysis described above, we formulated the following hypothesis.

(a) The value of the IDV-tt[n](x) increases as x increases, up to the limit value of x.

(b) The value of the IDV-tt[p](x) reaches its peak around 10 to the 5th power, and then decreases as x increases, and this also continues up to the limit value of x.

(c) When the values for α in the following two "li(x) -ali[α](x)" are the same, the value of the IDV-tt[n](x) by "li(x) -ali[α](x)" and the value of the IDV-tt[p](x) by "li(x) -ali[α](x)" converge as x increases, and this also continues up to the limit value of x.

(d) When we substitute the true value for α, the value to which both of the IDV-tt[n](x) by "li(x) -ali[α](x)" and the IDV-tt[p](x) by "li(x) -ali[α](x)" converge at the limit value of x is as follows.

This value is abbreviated as "C" or "C value" hereafter in this paper.

$$C = \frac{\pi^2}{10^{\frac{3}{2}}} \sim 0.312104295123$$

C value stands for the convergent value. In more detail, it is expressed as "the expected value to which the error rate of li(x) for π(x) converges."

Assuming that the hypothesis described above is true, the approximate value of the true value for α obtained using the method described below is 3.20405716 shown at the beginning of this paper. We created Tables 1 to 8 to verify the value.

Refinement of the true value for α through the hypothesis

Assuming that the hypothesis stated above is correct, let's refine the true value for α. The range of the true values for α before starting this work is more than 3.2, less than 3.21.

Pursuit of the true value for α using the IDV-tt[n](x): Assuming that the hypothesis is true, the following two points are important here. One is that the value of the IDV-tt[n](x) by "li(x) -ali[α](x)" continues to increase as x increases. The other is that when we substitute the true value for α, the value of the IDV-tt[n](x) by "li(x) -ali[α](x)" converges to the C value.

Therefore, the basic rule here is that when increasing x, we should not step over the C value. That is, if the value of the IDV-tt[n](x) by "li(x) -ali[α](x)" crosses the C value, at that point the value substituted to α is disqualified.

So let's start the pursuit.

Initially, we substitute 3.2 for α. Since the numbers up to the 100th power of 10 overlap with Table 21, the Table below starts from the 100th power of 10. Then, the IDV-tt[n](x) by "li(x) -ali[3.2](x)" crosses the C value at the trial of 10 to the 800th power, and is disqualified. After that, ali[3.201](x) and others take over, but candidates up to ali[3.2038](x) are disqualified in the trials up to 10 to the 14,000th power.

As a result, it become clear that the true value for α, given the hypothesis, is more than 3.2038 (Table 23).

Table 23. Table of IDV-tt[n](x) by "li(x) -ali[α](x)" with six different values for α. The Table uses IDV-tt[n](x) by six different "li(x) -ali[α](x)" to pursue the true value for α.

x	IDV-tt[n](x) by "li(x) -ali[α](x)"					
	α=3.2	α=3.201	α=3.202	α=3.203	α=3.2035	α=3.2038
10^{100}	0.309373467712	-	-	-	-	-
10^{120}	0.309894769991	-	-	-	-	-
10^{150}	0.310415986604	-	-	-	-	-
10^{200}	0.310937117732	-	-	-	-	-
10^{250}	0.311249755451	-	-	-	-	-
10^{300}	0.311458163555	-	-	-	-	-
10^{350}	0.311718654533	-	-	-	-	-
10^{400}	0.311874938914	-	-	-	-	-
10^{500}	0.311979124252	-	-	-	-	-
10^{600}	0.312109351146	0.312011847458	-	-	-	-
10^{800}	-	0.312089956633	-	-	-	-
10^{1000}	-	0.312142028356	0.312044544906	-	-	-
10^{1500}	-	-	0.312096599517	-	-	-
10^{2000}	-	-	0.312148653281	0.312051198193	-	-
10^{2500}	-	-	-	0.312082420294	-	-
10^{3000}	-	-	-	0.312103234858	-	-
10^{4000}	-	-	-	0.312129252873	0.312080535962	-
10^{5000}	-	-	-	-	0.312096144233	-
10^{6000}	-	-	-	-	0.312106549705	0.312077324421
10^{8000}	-	-	-	-	-	0.312090329995
10^{10000}	-	-	-	-	-	0.312098133314
10^{12000}	-	-	-	-	-	0.312103335517
10^{14000}	-	-	-	-	-	0.312107051370

Pursuit of the true value for α using IDV-tt[p](x): Assuming that the hypothesis is true, the following two points are important here. One is that the value of the IDV-tt[p](x) by "li(x) -ali[α](x)" reaches its peak around 10 to the fifth power, and after that, it continues to decrease, as x increases. The other is that when we substitute the true value for α, the value of the IDV-tt[p](x) by "li(x) -ali[α](x)" converges to the C value.

Therefore, the basic rule here is also that, although the direction is opposite to the IDV-tt[n](x), we should not step over the C value. That is, if the value of the IDV-tt[n](x) by "li(x) -ali[α](x)" crosses the C value, at that point the value substituted to α is disqualified.

So let's start the pursuit.

Initially, we substitute 3.21 for α. As already shown in Table 22, it crosses the C value at the trial of 10 to the 16th power. After that, as shown in the Table below, ali[3.209](x) and others take over, but candidates up to ali[3.206](x) are disqualified in the trials up to 10 to the 29th power.

As a result, we can see that the true value for α based on the hypothesis is less than 3.206 (Table 24).

Table 24. Table of the IDV-tt[p](x) by "li(x) -ali[α](x)" with six different values for α. The Table uses the IDV-tt[p](x) by six different "li(x) -ali[α](x)" to pursue the true value for α.

x	IDV-tt[p](x) of "li(x) -ali[α](x)"					
	α=3.205	α=3.206	α=3.207	α=3.208	α=3.209	α=3.21
10^{13}	0.312756007221	-	-	-	-	0.312402147289
10^{14}	0.312651072178	-	-	-	-	0.312269120785
10^{15}	0.312566926235	-	-	-	-	0.312164312000
10^{16}	0.312498417835	-	-	-	0.312177477661	0.312080267074
10^{17}	0.312441891773	-	-	-	0.312109034932	-
10^{18}	0.312394702753	-	-	0.312149804188	0.312052562960	-
10^{19}	0.312354898741	-	0.312199939557	0.312102648985	-	-
10^{20}	0.312321013244	-	0.312160154538	-	-	-
10^{21}	0.312291926676	-	0.312126285190	-	-	-
10^{22}	0.312266772671	0.312194539215	0.312097212469	-	-	-
10^{23}	0.312244872129	0.312169391192	-	-	-	-
10^{24}	0.312225686621	0.312147495854	-	-	-	-
10^{25}	0.312208784776	0.312128314900	-	-	-	-
10^{26}	0.312193817698	0.312111417065	-	-	-	-
10^{27}	0.312180500720	0.312096453535	-	-	-	-
10^{28}	0.312168599711	-	-	-	-	-
10^{29}	0.312889194370	-	-	-	-	-

Pursuit of the true value for α using IDV-tt[pp](x): Since the pursuit of the true value for α using the IDV-tt[p](x) reached a dead end at 10 to the power of 29, IDV-tt[pp](x) is used for further analysis. IDV-tt[pp] stands for intuitive distancing value at ten times by pseudo prime numbers.

The formula is as follows.

$$IDV-tt[pp](x)=\frac{\left(\dfrac{g(x)-f(x)}{g\left(\dfrac{x}{10}\right)-f\left(\dfrac{x}{10}\right)}\right)}{\left(\dfrac{ali[\alpha](x)}{ali[a]\left(\dfrac{x}{10}\right)}\right)}$$

It changes the "(π(x)/π(x/10))" part of the formula of IDV-tt[p](x) to "(ali[α](x)/ali[α](x/10))". Then, substitute the value being checked at that time for α. In short, the value of π(x) is estimated by ali[α](x) using the value for α that is being checked at the time.

Although we use the word "pseudo", as shown in the following Table, IDV-tt[pp](x) provides us a little more stable numbers than the IDV-tt[p](x), when x is larger than 10 to the 5th power (Tables 25).

Table 25. Table of "π(x)/π(x/10)" and "ali[α](x)/ali[α](x/10)". The Table shows "π(x)/π(x/10)," and "ali[α](x)/ali[α](x/10)" by three kinds of values for α.

x	π(x)/π(x/10)	ali[α](x)/ali[α](x/10)		
		α=3.2	α=3.205	α=3.21
10	--------	---------	--------	---------
10^2	6.25	6.932	6.924	6.916
10^3	6.72	6.326	6.325	6.324
10^4	7.315	7.185	7.184	7.183
10^5	7.804719	7.788709	7.788337	7.787967
10^6	8.183694	8.185756	8.185598	8.185442
10^7	8.466190	8.463398	8.463336	8.463274
10^8	8.669331	8.668259	8.668236	8.668212
10^9	8.825467	8.825320	8.825312	8.825303
10^{10}	8.949353	8.949342	8.949339	8.949336
10^{11}	9.049626	9.049662	9.049660	9.049659
10^{12}	9.132445712	9.132452211	9.132451825	9.132451447
10^{13}	9.201934334	9.201935108	9.201934975	9.201934845
10^{14}	9.261083262	9.261083539	9.261083494	9.261083449
10^{15}	9.312047689	9.312047821	9.312047806	9.312047791
10^{16}	9.356420182	9.356420186	9.356420181	9.356420176
10^{17}	9.395404470	9.395404462	9.395404460	9.395404458
10^{18}	9.429927690183	9.429927692306	9.429927691725	9.429927691160
10^{19}	9.460715430357	9.460715433019	9.460715432826	9.460715422638
10^{20}	9.488343741967	9.488343741073	9.488343741009	9.488343740947
10^{21}	9.513275847194	9.513275847200	9.513275847179	9.513275847159
10^{22}	9.535888526562	9.535888526601	9.535888526594	9.535888526587
10^{23}	9.556491394932	9.556491394945	9.556491394942	9.556491394940
10^{24}	9.575341251106526	9.575341251100657	9.575341251099911	9.575341251099192
10^{25}	9.592652890650813	9.592652890651551	9.592652890651307	9.592652890651073
10^{26}	9.608607364472738	9.608607364472711	9.608607364472631	9.608607364472555
10^{27}	9.623358360665339	9.623358360665410	9.623358360665384	9.623358360665359
10^{28}	9.637037189663467	9.637037189663460	9.637037189663451	9.637037189663443
10^{29}	9.649756716944958	9.649756716944964	9.649756716944961	9.649756716944959
10^{30}	-	9.661614493938594	9.661614493938593	9.661614493938592
10^{31}	-	9.672695271711581400	9.672695271711581114	9.672695271711580841
10^{32}	-	9.683073034973006405	9.683073034973006313	9.683073034973006225
10^{33}	-	9.692812659959087272	9.692812659959087242	9.622812659959087214
10^{34}	-	9.701971274961307348	9.701971274961307338	9.701971274961307329

The pursuit of the true value for α using the IDV-tt[pp](x) begins with ali[3.205](x), which has passed the trial of 10 to the 29th power. However, ali[3.205](x) crosses the C value at the 10 to the 40th power trial and is disqualified. After that, up to ali[3.204057162](x) crosses the C value and are disqualified, in the trials up to 10 to the power of 14,000. In detail, the seven values for α in the Table below are the smallest decimal numbers with three to nine decimal places that result in disqualification in the trials up to 10 to the power of 14,000.

As a result, we can see that the true value for α based on the hypothesis is less than 3.204057162 (Table 26).

Table 26. Table of the IDV-tt[pp](x) by "li(x) -ali[α](x)" with seven different values for α. The Table uses the IDV-tt[pp](x) by seven different "li(x) -ali[α](x)" to pursue the true value for α.

x	IDV-tt[pp](x) by "li(x) -ali[α](x)"						
	α=3.204057162	α=3.20405717	α=3.2040572	α=3.204058	α=3.20406	α=3.2041	α=3.205
10^{30}	-	-	-	-	-	-	0.312157920669
10^{31}	-	-	-	-	-	-	0.312148301741
10^{32}	-	-	-	-	-	-	0.312139607025
10^{33}	-	-	-	-	-	-	0.312131721732
10^{34}	-	-	-	-	-	-	0.312124548369
10^{35}	-	-	-	-	-	-	0.312118003698
10^{40}	-	-	-	-	-	0.312180183198	0.312092525055
10^{50}	-	-	-	-	-	0.312150707416	-
10^{60}	-	-	-	-	-	0.312134954787	-
10^{80}	-	-	-	-	-	0.312119511373	-
10^{100}	-	-	-	-	-	0.312112454141	-
10^{120}	-	-	-	-	-	0.312108650656	-

10^{150}	-	-	-	-	-	0.312105557876	-
10^{200}	-	-	-	-	0.312107063421	0.312103167085	-
10^{250}	-	-	-	-	0.312105962323	-	-
10^{300}	-	-	-	-	0.312105366039	-	-
10^{400}	-	-	-	-	0.312104774748	-	-
10^{500}	-	-	-	-	0.312104501741	-	-
10^{600}	-	-	-	-	0.312104353668	-	-
10^{800}	-	-	-	0.312104401452	0.312104206634	-	-
10^{1000}	-	-	-	0.312104333481	-	-	-
10^{1200}	-	-	-	0.312104296586	-	-	-
10^{1500}	-	-	0.312104344345	0.312104266418	-	-	-
10^{2000}	-	-	0.312104320895	-	-	-	-
10^{2500}	-	-	0.312104310046	-	-	-	-
10^{3000}	-	-	0.312104304155	-	-	-	-
10^{4000}	-	-	0.312104298298	-	-	-	-
10^{5000}	-	-	0.312104295588	-	-	-	-
10^{6000}	-	0.312104297039	0.312104294117	-	-	-	-
10^{8000}	-	0.312104295576	-	-	-	-	-
10^{10000}	0.312104295678	0.312104294898	-	-	-	-	-
10^{12000}	0.312104295310	-	-	-	-	-	-
10^{14000}	0.312104295088	-	-	-	-	-	-

Further refinement of the true value for α by ARC-sq(x)

If we based on the pursuit above, the range of the true value for α is more than 3.2038, less than 3.204057162. From here, let's further narrow down it using ARC-sq(x).

ARC-sq stands for the approach rate for the C value at square. There are two types of ARC-sq(x): ARC-sq[n](x) based on the IDV-tt[n](x) and ARC-sq[pp](x) based on the IDV-tt[pp](x).

Basic concepts: ARC-sq(x) is a tool for identifying that, when a certain number is substituted for α, of which value, IDV-tt[n](x) or IDV-tt[pp](x), cross the C value, on the way to the limit value of x. This value is derived from VLC(x) and AVC-sq(x), as explained later.

Contents of VLC(x): VLC stands for the value left to the C value. VLC(x) means the difference between the IDV-tt(x) and the C value. There are two types of VLC(x): VLC[n](x) based on the IDV-tt[n](x) and VLC[pp](x) based on the IDV-tt[pp](x).

The formula for each is as follows:

$$VLC[pp](x) = IDV\text{-}tt[pp](x)\text{-}C$$

VLC(x) decreases as x increases, and if it becomes a negative number, it ends there. This is because that is where the value of the IDV-tt(x) crosses the C value.

Contents of AVC-sq(x): AVC-sq stands for the approach value for the C value at square. AVC-sq(x) means the numerical reduction of the VLC(x) when the value of x is squared. There are also two types of AVC-sq(x): AVC-sq[n](x) based on the VLC[n](x) and AVC-sq[pp](x) based on the VLC[pp](x).

The formula for each is as follows:

$$AVC\text{-}sq[n](x) = VLC[n]\left(\sqrt{x}\right)\text{-}VLC[n](x)$$

$$AVC\text{-}sq[pp](x) = VLC[pp]\left(\sqrt{x}\right)\text{-}VLC[pp](x)$$

Contents of ARC-sq(x): ARC-sq (x) stands for the approach rate for the C value at square. ARC-sq(x) means the reduction rate of the VLC(x) when the value of x is squared. As already mentioned, there are two types of ARC-sq(x). Using a different expression from before, they become as follows. One is ARC-sq[n](x), which is based on the VLC[n](x) and the AVC-sq[n](x). The other is ARC-sq[pp](x), which is based on the VLC[pp](x) and the AVC-sq[pp](x).

The formula for each is as follows:

$$ARC-sq[n](x)=\frac{AVC-sq[n](x)}{VLC[n]\left(\sqrt{x}\right)}$$

$$ARC-sq[pp](x)=\frac{AVC-sq[pp](x)}{VLC[pp]\left(\sqrt{x}\right)}$$

As shown in Table 27, when the value of the IDV-tt[n](x) by "li(x) -ali[α](x)" crosses the C value, it is the case which the ARC-sq[n](x) exceeds 1. Conversely, as shown in Table 28, when the value of the IDV-tt[pp](x) by "li(x) -ali[α](x)" crosses the C value, it is the case which the ARC-sq[pp](x) exceeds 1.

However, when the values for α in both of following "li(x) -ali[α](x)" are equal, the values of both IDV-tt[n](x) by "li(x) -ali[α](x)" and IDV-tt[pp](x) by "li(x) -ali[α](x)" never cross the C value. Therefore, when the values of α in both of following "li(x) -ali[α](x)" are equal, the values of both of ARC-sq[n](x) based on the IDV-tt[n](x) by "li(x) -ali[α](x)" and ARC-sq[pp](x) based on the IDV-tt[pp](x) by "li(x) -ali[α](x)"cannot exceed 1.

The bottom value for the rebounding: The common feature of the ARC-sq[n](x) and the ARC-sq[pp](x), of which value exceeds 1 on the way to the limit value of x, is that it begins to rise after a long decline. However, the difference is in the bottom value.

The ARC-sq[n](x) falls to just above 0.5 before beginning to rise. On the other hand, the ARC-sq[pp](x) falls to just above 0.75 before starting to rise.

The following two Tables are good examples to see the difference in bottom values. Table 27 shows the progress of the ARC-sq[n](x) regarding the IDV-tt[n](x) by "li(x) -ali[3.2038](x)", which crosses the C value at the trial of 10 to the 14,000th power. Table 28 shows the progress of the ARC-sq[pp](x) regarding the IDV-tt[pp](x) by "li(x) -ali[3.204057162](x)", which similarly crosses the C value at the trial of 10 to the 14,000th power (Tables 27 and 28).

Table 27. Table of the ARC-sq[n](x) regarding the IDV-tt[n](x) by "li(x) -ali[3.2038](x)". The Table shows the progress of the ARC-sq[n](x) regarding the IDV-tt[n](x) by "li(x) -ali[3.2038](x)."

x	li(x) -ali[3.2038](x)			
	IDV-tt[n](x)	VLC[n](x)	AVC-sq[n](x)	ARC-sq[n](x)
10^{2}	0.134029869392	0.178074425731	-----	-----
10^{4}	0.232463052930	0.079641242193	0.098433183538	0.552764290177
10^{8}	0.272859383038	0.039244912085	0.040396330108	0.507228779910
10^{14}	0.289754369903	0.022349925219	-----	-----
10^{16}	0.292560242342	0.019544052781	0.019700859304	0.501997794299
10^{18}	0.294740763634	0.017363531489	-----	-----
10^{20}	0.296484043563	0.015620251560	-----	-----
10^{28}	0.300962179432	0.011142115691	0.011207809528	0.501469665707
10^{32}	0.302360258066	0.009744037057	0.009800015724	0.501432115139
10^{36}	0.303447216162	0.008657078960	0.008706452529	0.501421760552
10^{40}	0.304316509007	0.007787786115	0.007832465445	0.501430173182
10^{64}	0.307248588871	0.004855706252	0.004888330805	0.501674077830
10^{125}	0.309631335278	0.002472959845	-----	-----
10^{128}	0.309689904834	0.002414390289	0.002441315963	0.502772580598
10^{250}	0.310880585336	0.001223709786	0.001249250059	0.505163907618
10^{500}	0.311505026839	0.000599268283	0.000624441503	0.510285616783
10^{1000}	0.311817201844	0.000287093279	0.000312175004	0.520926959060
10^{1500}	0.311921253411	0.000183041712	-----	-----
10^{2000}	0.311973277926	0.000131017197	0.000156076082	0.543642410648
10^{2500}	0.312004492229	0.000099802893	-----	-----
10^{3000}	0.312025301596	0.000078993527	0.000104048185	0.568439751913
10^{4000}	0.312051313114	0.000052982008	0.000078035188	0.595610272212
10^{5000}	0.312066919924	0.000037375199	0.000062427694	0.625509864356
10^{6000}	0.312077324421	0.000026970702	0.000052022825	0.658570736483
10^{7000}	0.312084756184	0.000019538938	-----	-----
10^{8000}	0.312090329995	0.000013965127	0.000039016881	0.736417554188
10^{10000}	0.312098133314	0.000006161808	0.000031213391	0.835136444733
10^{12000}	0.312103335517	0.000000959606	0.000026011096	0.964420430475
10^{14000}	0.312107051370	-0.000002756247	0.000022295186	1.141064348109

Table 28. Table of the ARC-sq[pp](x) regarding the IDV-tt[pp](x) by "li(x) -ali[3.204057162](x)". The Table shows the progress of the ARC-sq[pp](x) regarding the IDV-tt[pp](x) by "li(x) -ali[3.204057162](x)."

x	li(x) -ali[3.204057162](x)			
	IDV-tt[pp](x)	**VLC[pp](x)**	**AVC-sq[pp](x)**	**ARC-sq[pp](x)**
10^{2}	0.193524129644	-0.118580165478	-----	-----
10^{4}	0.323554103557	0.011449808435	-0.130029973913	1.096557534631
10^{8}	0.314755389642	0.002651094519	0.008798713916	0.768459486980
10^{16}	0.312658864492	0.000554569369	0.002096525150	0.790814938736
10^{32}	0.312231449209	0.000127154086	0.000427415283	0.770715634608
10^{64}	0.312134829456	0.000030534333	0.000096619753	0.759863531987
10^{125}	0.312112148358	0.000007853236	-----	-----
10^{128}	0.312111781059	0.000007485937	0.000023048396	0.754835426891
10^{250}	0.312106238771	0.000001943648	0.000005909588	0.752503547839
10^{500}	0.312104778188	0.000000483065	0.000001460583	0.751464756634
10^{1000}	0.312104415110	0.000000119987	0.000000363078	0.751613428370
10^{1500}	0.312104348046	0.000000052924	-----	-----
10^{2000}	0.312104324596	0.000000029474	0.000000090513	0.754359106945
10^{2500}	0.312104313748	0.000000018625	-----	-----
10^{3000}	0.312104307856	0.000000012734	0.000000040190	0.759397690766
10^{4000}	0.312104302000	0.000000006877	0.000000022596	0.766663023255
10^{5000}	0.312104299290	0.000000004167	0.000000014458	0.776249726519
10^{6000}	0.312104297818	0.000000002695	0.000000010038	0.788317396587
10^{7000}	0.312104296931	0.000000001808	-----	-----
10^{8000}	0.312104296355	0.000000001232	0.000000005645	0.820832934504
10^{10000}	0.312104295678	0.000000000555	0.000000003612	0.866828121635
10^{12000}	0.312104295310	0.000000000187	0.000000002508	0.930576096142
10^{14000}	0.312104295088	-0.000000000035	0.000000001843	1.019166175369

The result of the analysis by the ARC-sq(x): The analysis using the ARC-sq(x) to eight decimal places shows that the true value for α is more than 3.20405715 and less than 3.20405716.

The reason for more than 3.20405715 is, on the one hand, as shown in Table 29, the ARC-sq[n](x) regarding the IDV-tt[n](x) by "li(x) -ali[3.20405715](x)" shows rebound from the trial of 10 to the 10,000th power. On the other hand, as shown in Table 30, the ARC-sq[pp](x) regarding the IDV-tt[pp](x) by "li(x) -ali[3.20405715](x)" traces a downward trend, straddling the bottom value for the rebounding, at the trial of 10 to the 1,000th power.

When α is 3.20405715 and x is 10 to the power of 14,000, the value of the ARC-sq[n](x) is less than the value of the ARC-sq[pp](x). Nevertheless, for the reasons described above, when α is 3.20405715, it is the ARC-sq[n](x) that exceeds 1.

The reason for less than 3.20405716 is that, on the one hand, as shown in Table 30, the ARC-sq[pp](x) regarding the IDV-tt[pp](x) by "li(x) -ali[3.20405716](x)" shows rebound from the trial of 10 to the 2,000th power. It's on track to reach 1 at around 10 to the power of 16,000. On the other hand, as shown in Table 29, the ARC-sq[n](x) regarding the IDV-tt[n](x) by "li(x) -ali[3.20405716](x)" traces a downward trend, straddling the bottom value for the rebounding, at the trial of 10 to the 10,000th power. Therefore, when α is 3.20405716, it is the ARC-sq[pp](x) that exceeds 1.

In this way, according to the pursuit up to the 8th decimal place, the range of the true value for α based on the hypothesis is more than 3.20405715, less than 3.20405716.

Therefore, in order to determine whether the approximate value for α to the 8th decimal place is 3.20405715 or 3.20405716, we performed an analysis of the ARC-sq(x) regarding the IDV-tt(x) by "li(x) -ali[3.204057155](x)". As a result, we can see that the true value for α is more than 3.204057155. The reason is as follows. On the one hand, neither the ARC-sq[n](x) nor the ARC-sq[pp](x) shows rebounding at this stage. But on the other hand, the latter crosses the bottom value for the rebounding at the trial of 10 to the 3,000th power. Therefore it is the former that rebounds.

As a result, the range of the true values for α based on the hypothesis is more than 3.204057155, less than 3.20405716. Therefore, when creating Tables 1 to 8, we adopted 3.20405716 as an approximate value for α (Tables 29 and 30) [1-6].

Table 29. Table of the ARC-sq[n](x) regarding the IDV-tt[n](x) by "li(x) -ali[α](x)". The Table compares the ARC-sq[n](x) regarding the IDV-tt[n](x) by "li(x) -ali[α](x)" with three different α.

x	ARC-sq[n](x) regarding IDV-tt[n](x) by "li(x) -ali[α](x)"		
	α=3.20405715	α=3.204057155	α=3.20405716
10^{4}	0.552679817586	0.552679815944	0.552679814301
10^{8}	0.507067824275	0.507067821146	0.507067818018
10^{16}	0.501677273113	0.501677266885	0.501677260656
10^{32}	0.500790093932	0.500790081464	0.500790068997
10^{64}	0.500387578086	0.500387553135	0.500387528185
10^{128}	0.500192041261	0.500191991343	0.500191941424
10^{250}	0.500097965036	0.500097867522	0.500097770008

10^{500}	0.500049025381	0.500048830335	0.500048635288
10^{1000}	0.500024796980	0.500024406868	0.500024016757
10^{2000}	0.500013016423	0.500012236179	0.500011455939
10^{3000}	0.500009370460	0.500008200085	0.500007029715
10^{4000}	0.500007756389	0.500006195881	0.500004635382
10^{5000}	0.500006954793	0.500005004151	0.500003053525
10^{6000}	0.500006559357	0.500004218581	0.500001877828
10^{8000}	0.500006377639	0.500003256593	0.500000135586
10^{10000}	0.500006601978	0.500002700659	0.499998799401
10^{12000}	0.500007029328	0.500002347733	0.499997666225
10^{14000}	0.500007572678	0.500002110804	0.499996649049

Table 30. Table of the ARC-sq[pp](x) regarding the IDV-tt[pp](x) by "li(x) -ali[α](x)". The Table compares the ARC-sq[pp](x) regarding the IDV-tt[pp](x) by "li(x) -ali[α](x)" with three different α.

x	ARC-sq[pp](x) regarding IDV-tt[pp](x) by "li(x) -ali[α](x)"		
	α=3.20405715	α=3.204057155	α=3.20405716
10^{4}	1.096557543935	1.096557540059	1.096557536182
10^{8}	0.768459406771	0.768459440192	0.768459473612
10^{16}	0.790814589896	0.790814735246	0.790814880596
10^{32}	0.770714010120	0.770714686989	0.770715363860
10^{64}	0.759856546739	0.759859457244	0.759862367770
10^{128}	0.754806531543	0.754818571003	0.754830610846
10^{250}	0.752391558670	0.752438216772	0.752484880661
10^{500}	0.751013097660	0.751201222902	0.751389442416
10^{1000}	0.749799082604	0.750553994422	0.751310427890
10^{2000}	0.747081061471	0.750096444544	0.753136267755
10^{3000}	0.742987599964	0.749738164432	0.756612520875
10^{4000}	0.737417473202	0.749327582703	0.761628730840
10^{5000}	0.730408954258	0.748834704809	0.768214152796
10^{6000}	0.722036443385	0.748249822951	0.776438248463
10^{8000}	0.701586822766	0.746790858002	0.798221152752
10^{10000}	0.676948658933	0.744940249409	0.828114704465
10^{12000}	0.649093422558	0.742698912651	0.867851398571
10^{14000}	0.618994087990	0.740071866657	0.920034354133

Conclusion

We went through a lot of trial and error in our quest for the truth of the prime number theorem. By searching a law regarding x' such that li(x') equals π(x), we reached the prototype of ali(x). It is as follows.

$$li\left(x'\right)=li\left(\left(1-\left(\frac{1}{\alpha-\frac{1}{\log_{10}x}}\right)^{\log_{10}x}\right)x\right)=\pi\left(x\right)$$

We confirmed the existence of the law by observing that the value for α in this formula converges to around 3.2 as x increases.

After reaching the basic form of ali(x), we worked on refining the true value for α. Initially, we selected 3.2, 3.205, and 3.21 as tentative candidates for the true value for α and compared them. As a result, we found that the range of the true value for α is more than 3.2, less than 3.21, and around 3.205.

Based on these results, we performed an analysis using the two types of the IDV-tt(x) and found that the range of the true value for α is where the limit value of the IDV-tt(x) become more than 0.311981279099, less than 0.312168599711.

Here, we formulated a hypothesis. It is that when you substitute the true value for α, the limit value of the IDV-tt(x) converges to the following value. We named it "C" or the "C value."

$$C=\frac{\pi^2}{10^{\frac{3}{2}}}\sim 0.312104295123$$

The meaning of the C value is "the expected value to which the error rate of li(x) for π(x) converges."

After formulating the hypothesis, we further pursued the true value for α using the C value as an indicator. As a result of pursuit using the three types of the IDV-tt(x) and the two types of the ARC-sq(x), we obtained the true value for α, approximated to eight decimal places, as follows:

$$\alpha\sim 3.20405716$$

Then, we substituted this value into α of ali(x) below and verified it.

$$ali(x) = li\left(\left(1 - \left(\frac{1}{\alpha - \frac{1}{\log_{10} x}}\right)^{\log_{10} x}\right)x\right) \sim \pi(x)$$

This value requires verification. The reason for it is that there is a kind of weakness in the scientific basis for the value that is considered to be the true value for α. That is, the value has the condition that "when the hypothesis is true." And that hypothesis is based on the C value, which is obtained through what can be called devine revelation, intuitive inspiration, or instinctive inspiration.

Therefore, verification for this value is essential.

The results of the verification are shown in Tables 1-8. For all powers of 10, from 10 squared to 10 raised to the 8^{th} power, the value of ali(x) exceeds the value of π(x). To avoid any misunderstanding, we have provided the "closest reversals" for Tables 2-7.

When verified in this way, ali(x) leaves results worthy of publication, regardless of whether it passes through the true center for the scattering of the prime counting functions. On the other hand, as x becomes larger, the absolute value of the error of ali(x) for π(x), whether positive or negative, tends to increase. Therefore, we introduced the corridors for the prime number theorem to show that π(x) converges as x increases.

Each corridor becomes broader as x increases. This causes a larger proportion of π(x) to gather in ac[3.20](x) which we have named the main corridor as x becomes larger.

On the one hand, we should admit that the number of numbers we have investigated is too much small for verification, given the world of huge numbers. On the other hand, this is a study of the distribution of the prime numbers using a kind of epidemiological statistical methods. Since this is not a so-called mathematical proof, it is true that a sort of risks have remained.

To be honest, our methods cannot handle new mutations. For example, "x/log (x/e+e^2)" performs very well as an approximation of π(x) from 2 to around 3,500, or around 59 squared. However, after all, this formula becomes coproportionalize in the relation to π(x) or in the relation to ali(x).

In this way, although it is very modest compared to the biological world, mutations do occur even in the world of the distribution of the prime numbers. In short, at the stage of this paper, ali(x) is a prediction, not a guarantee, regarding the distribution of the prime numbers beyond 10 to the 30^{th} power.

But even so, confidence in this prediction will grow as the year progresses. Before long, proof of the prime number theorem will mean "proof of whether ali(x) keeps converging to π(x) up to the limit value of x." And the predictions presented in this paper will be looked upon with surprise for hundreds of years, whenever a new prime counting function for larger numbers is announced.

Finally, we make a declaration.

"We can see that ali(x) is truly an advanced conjecture for the prime counting functions. The main corridor is the very galaxy for the prime number theorem."

Acknowledgement

None.

Conflict of Interest

None.

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